Building Phylogenetic Networks

Josh Collins

November 10, 2011

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- For crossover operator require a function capable of combining two phylogenetic networks.
- For GAs don't want to be solving $\mathcal{NP}$-hard problems as part of algorithm.
- Want to ideally be at least $\mathcal{P}$.

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Construct a phylogenetic network that displays a pair of phylogenetic networks.

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Construct a phylogenetic network that displays a pair of phylogenetic networks.

Aim'

Construct a phylogenetic network that displays a set of phylogenetic trees.

Maximal Acyclic Agreement Forests

- An acyclic agreement forest $\mathcal{F}$ for a set of phylogenetic trees $\mathscr{T}$ is a collection of trees such that
  - Each tree in $\mathcal{F}$ is a subtree of all trees in $\mathscr{T}$,
  - No pair of trees in $\mathcal{F}$ overlap in any tree in $\mathscr{T}$.
  - For a pair of trees $\mathcal{T}_i, \mathcal{T}_j \in \mathcal{F}$ if there is a path in a tree in $\mathscr{T}$ from the most recent common ancestor (MRCA) of $\mathcal{T}_i$ to the MRCA of $\mathcal{T}_j$ then there is no tree in $\mathscr{T}$ such that there is a path in the opposite direction.

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- Going to make use of forests as a way to create an upper bound on hybridisation events of constructed network.
- Calculating a maximum acyclic agreement forest for a pair of trees is $\mathcal{NP}$-hard. (Bordewich & Semple, 2005)
- Is there a weakening of maximum acyclic agreement forests such that one can be calculated in $\mathcal{P}$ time?

Maximal Acyclic Agreement Forests

Definition

Let $\mathcal{F}$ and $\mathcal{F}'$ be two forests. Call $\mathcal{F}'$ a *subforest* of $\mathcal{F}$ if the members of $\mathcal{F}'$ are non-overlapping subtrees of $\mathcal{F}$.

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Theorem

Calculating a maximal acyclic agreement forest takes polynomial time.

$\mathcal{F} \leftarrow \{\text{'}\ell\text{'} : \ell \in L(\mathscr{T})\}$; $\{\ell_1, \ell_2, \ldots, \ell_n\} \leftarrow L(\mathcal{F})$;
for $i = 1, \ldots, n$ **do**

  for $j = i + 1, \ldots, n$ **do**
    $\mathcal{T}_1 \leftarrow$ The tree $\mathcal{T}_1 \in \mathcal{F}$ such that $\ell_i \in L(\mathcal{T}_1)$;
    $\mathcal{T}_2 \leftarrow$ The tree $\mathcal{T}_2 \in \mathcal{F}$ such that $\ell_j \in L(\mathcal{T}_2)$;
    $L_1 \leftarrow L(\mathcal{T}_1)$; $L_2 \leftarrow L(\mathcal{T}_2)$;
    $\mathcal{T} :\in \mathscr{T}$; $\mathcal{T}' \leftarrow \mathcal{T} \mid (L_1 \cup L_2)$;
    if $\neg$ isSubtree$(\mathcal{T}_1, \mathcal{T}_2, \mathscr{T})$ **then continue**;
    if isOverlap$(\mathcal{T}', \mathcal{F} \setminus \{\mathcal{T}_1, \mathcal{T}_2\}, \mathscr{T})$ **then continue**;
    if IsCyclic$(\mathcal{T}', \mathcal{F} \setminus \{\mathcal{T}_1, \mathcal{T}_2\}, \mathscr{T})$ **then continue**;
    else $\mathcal{F} \leftarrow \{\mathcal{T}'\} \cup \mathcal{F} \setminus \{\mathcal{T}_1, \mathcal{T}_2\}$;
  end
end
return $\mathcal{F}$;

Function findMaximalForest$(\mathscr{T})$

How long does it take to find a maximal acyclic agreement forest?

$$\textsc{findMaximalForest}(\mathscr{T}) \in \mathcal{O}(|X|^5 \, |\mathscr{T}|)$$

Construction

- Input now consists of a set of phylogenetic trees $\mathscr{T}$ and a phylogenetic forest $\mathcal{F}$.

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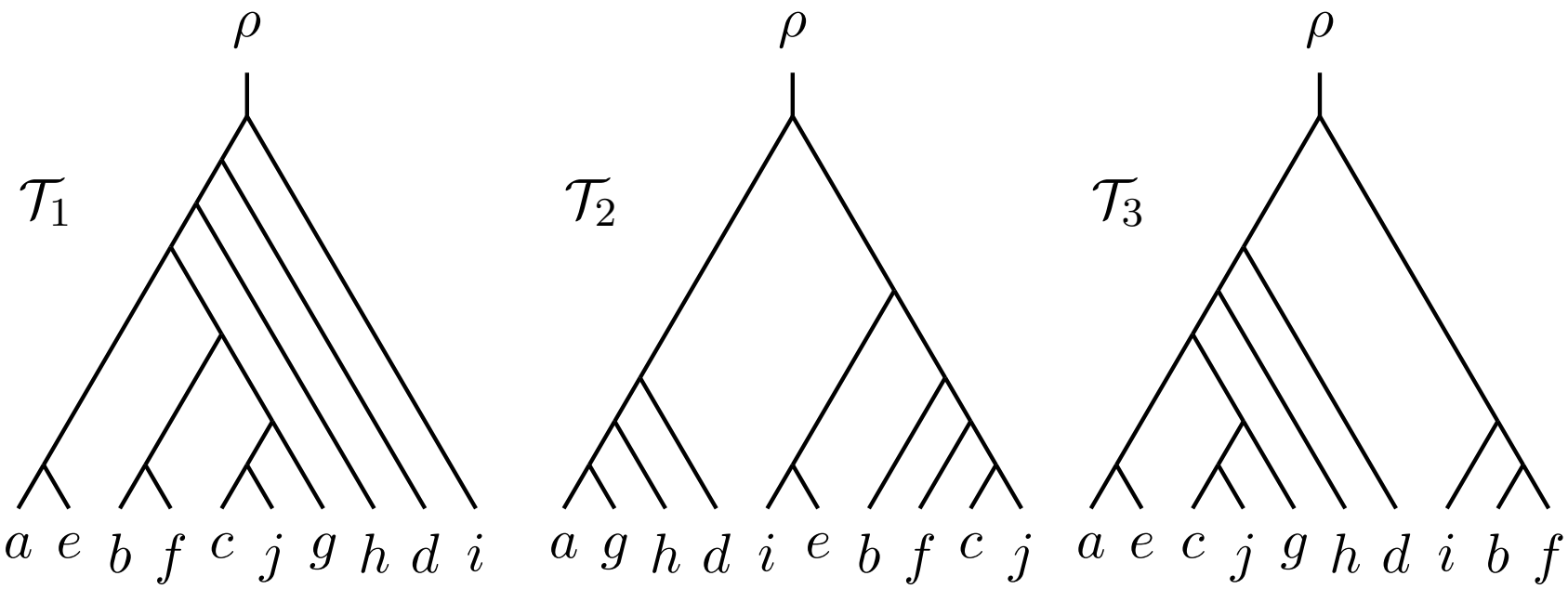
- Input now consists of a set of phylogenetic trees $\mathscr{T}$ and a phylogenetic forest $\mathcal{F}$.
- Output a phylogenetic network $\mathcal{N}$ such that
  - $h(\mathscr{T}) \leq h(\mathcal{N}) \leq |\mathcal{F}|\,|\mathscr{T}|$
  - $\mathcal{N}$ displays $\mathscr{T}$, that is there is some deletion of the egdes of $\mathcal{N}$ that gives $\mathcal{T}$ for each $\mathcal{T} \in \mathscr{T}$.
  - The forest induced by $\mathcal{N}$ is a superforest of $\mathcal{F}$, that is $\mathcal{F}$ is a subforest of the forest obtained by deleting all the in-arcs of vertices with in-degree greater than or equal to two.

Example



$$\mathcal{F} = \left\{ \rho \atop a\ g\ h\ d\ i \;,\; \wedge \atop b\ f \;,\; \wedge \atop c\ j \;,\; \bullet \atop e \right\}$$

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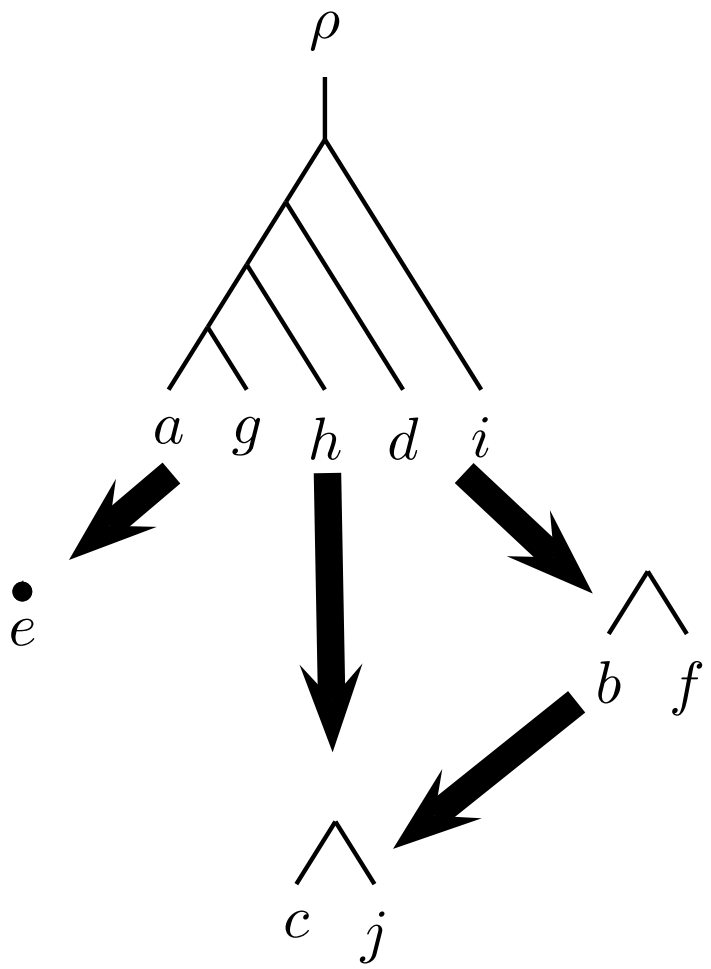

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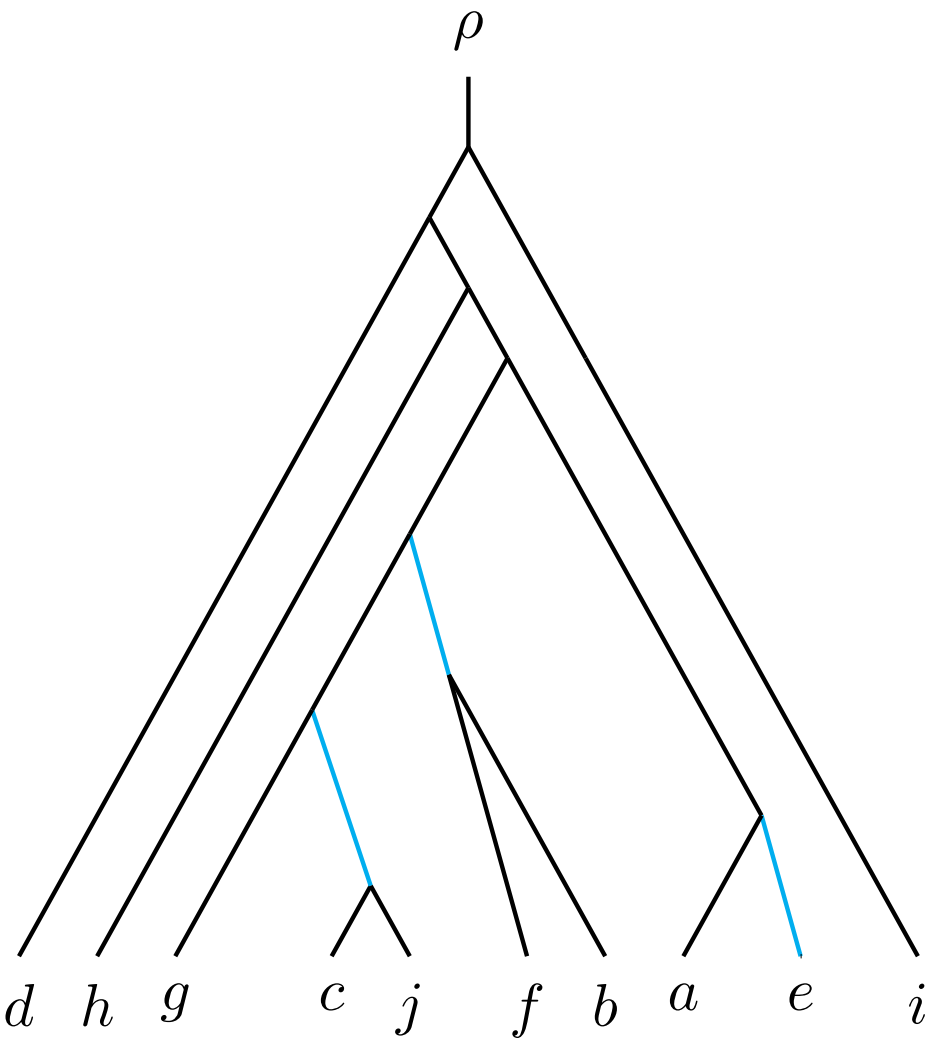

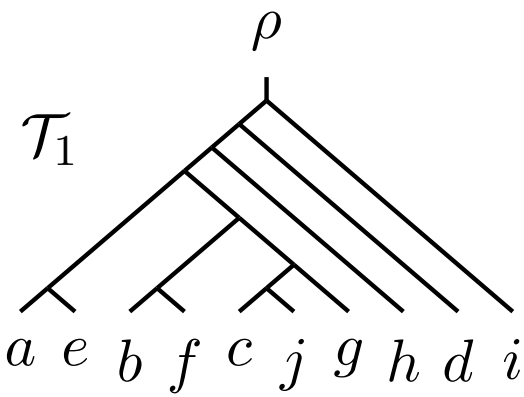

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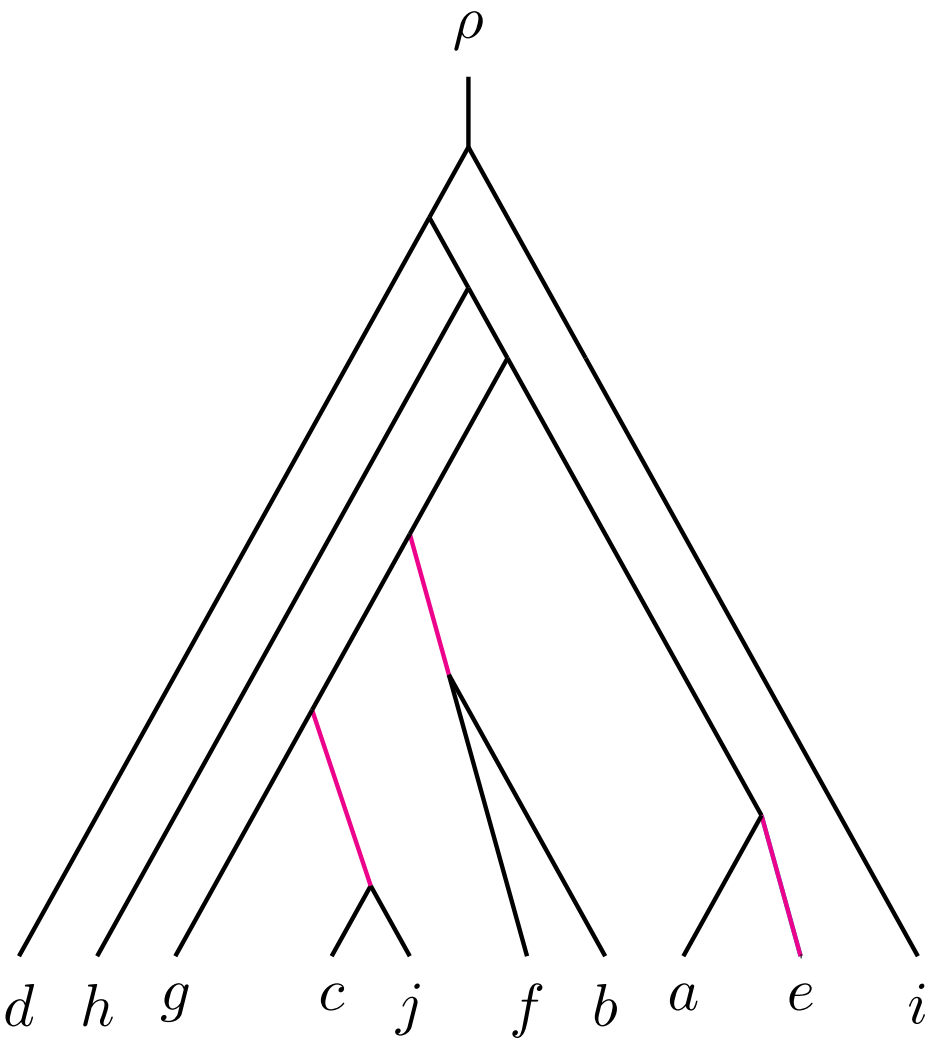

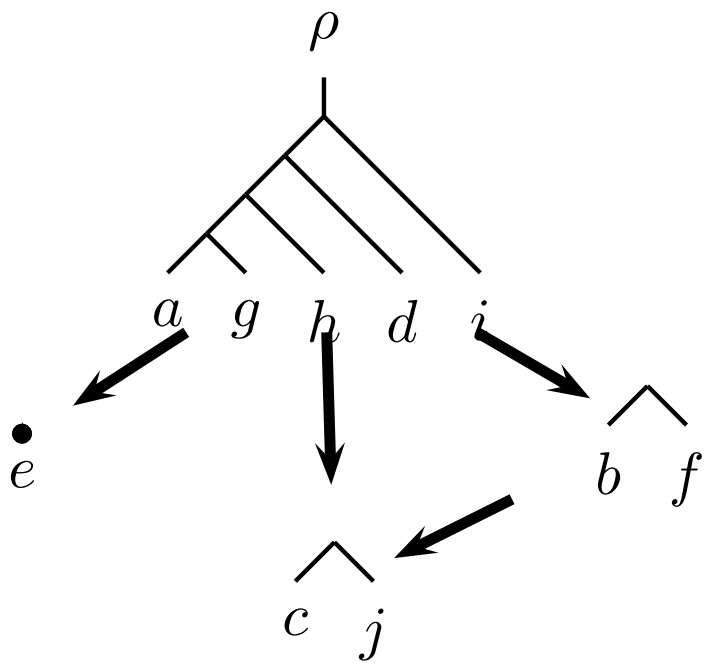

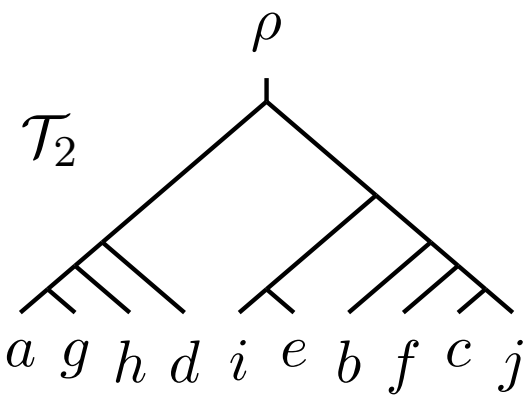

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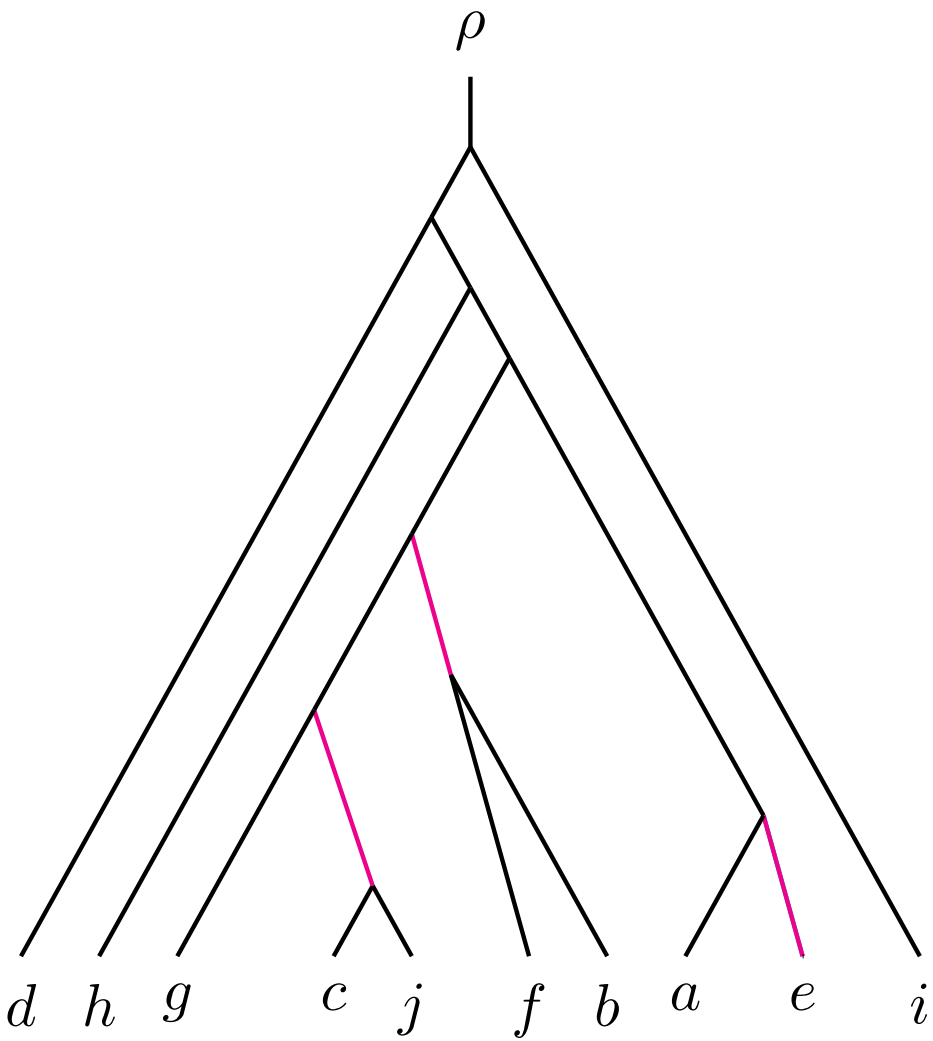

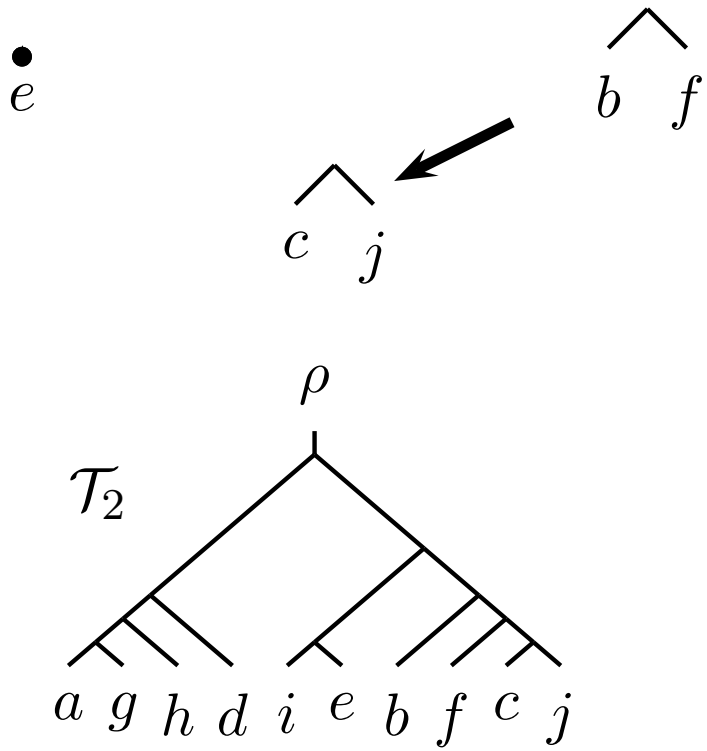

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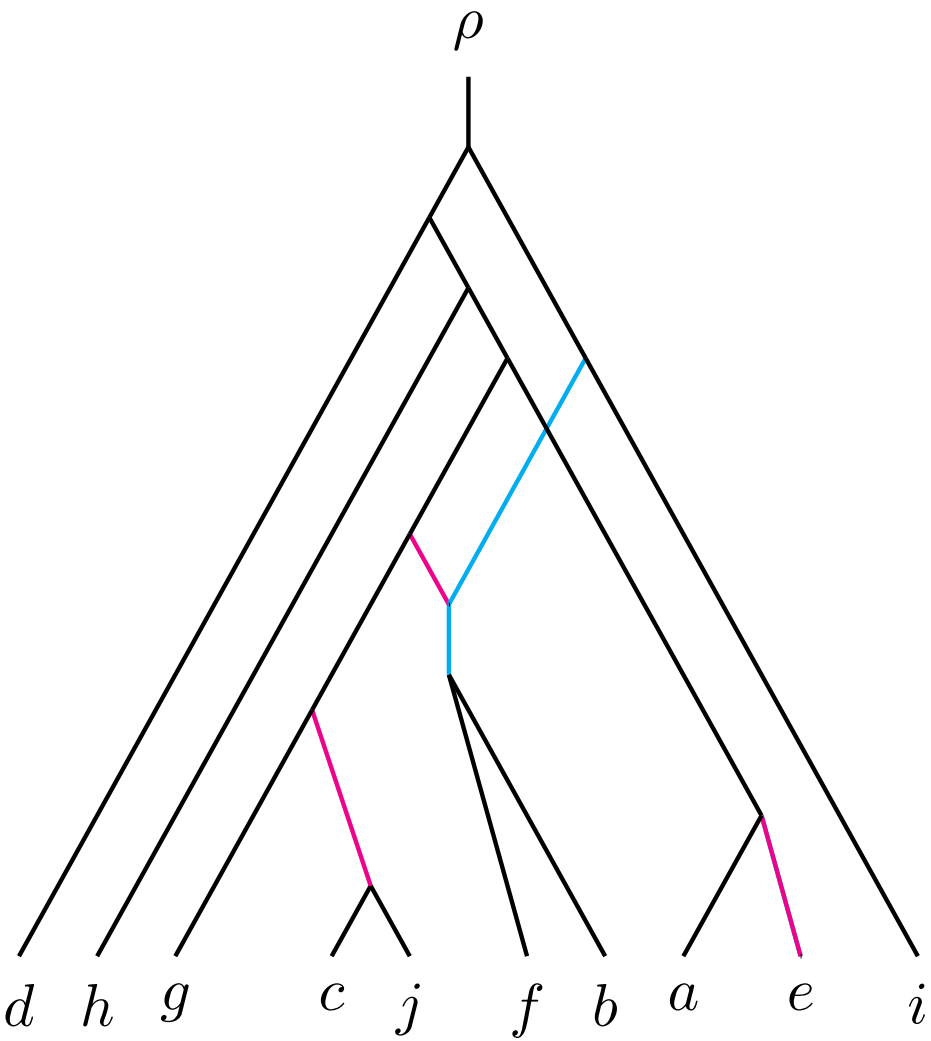

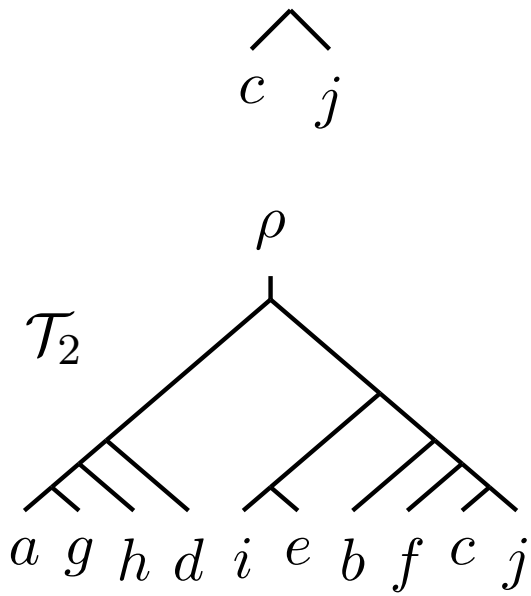

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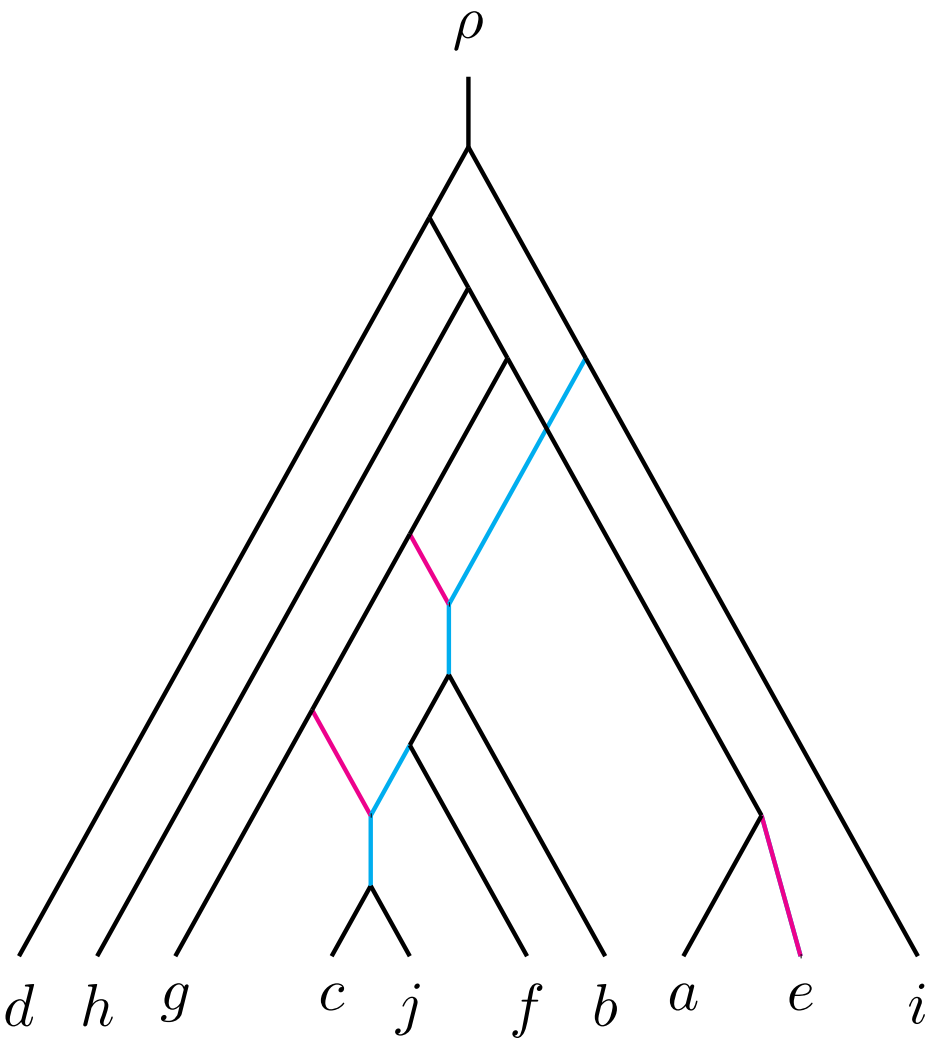

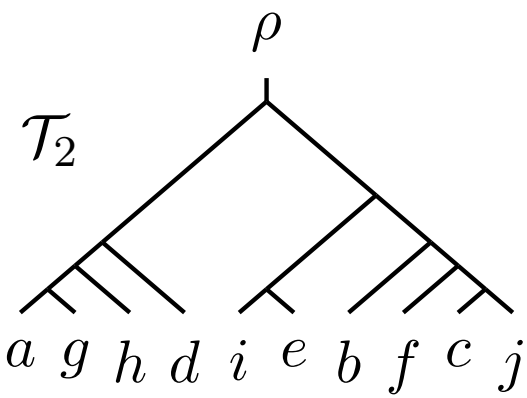

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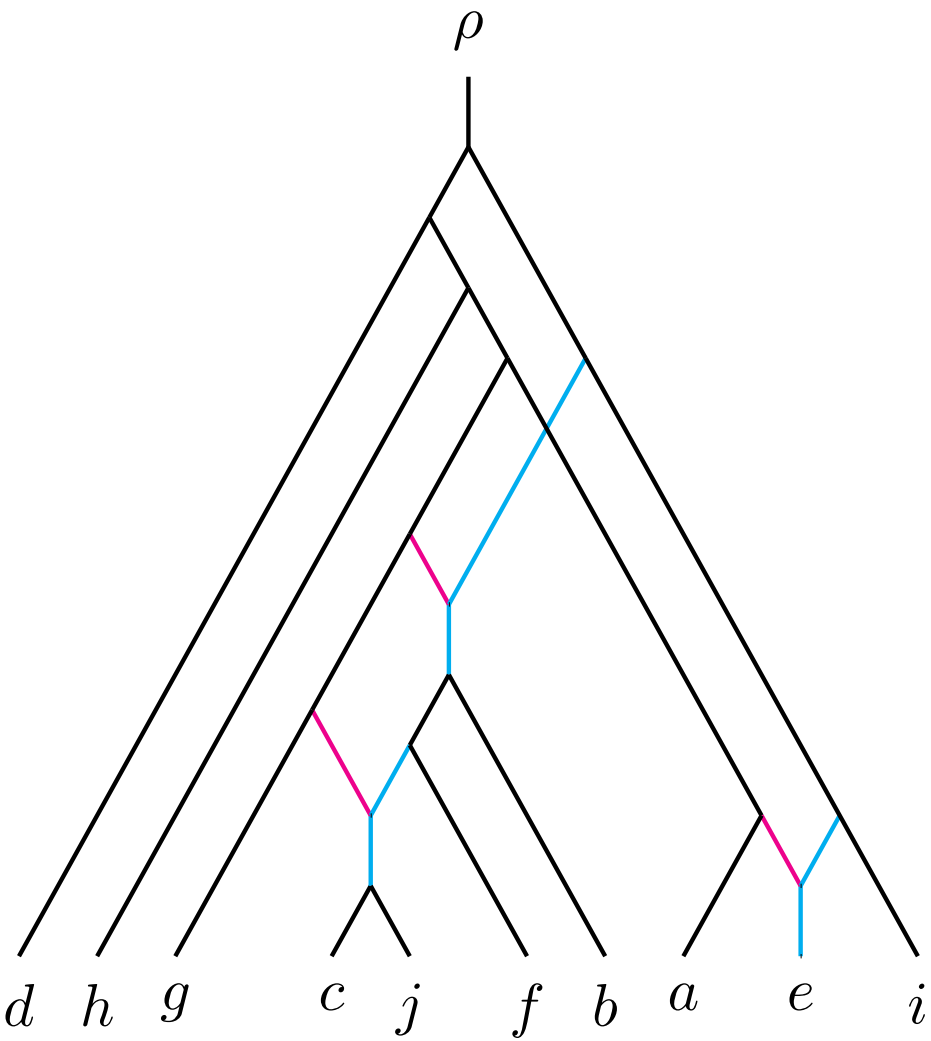

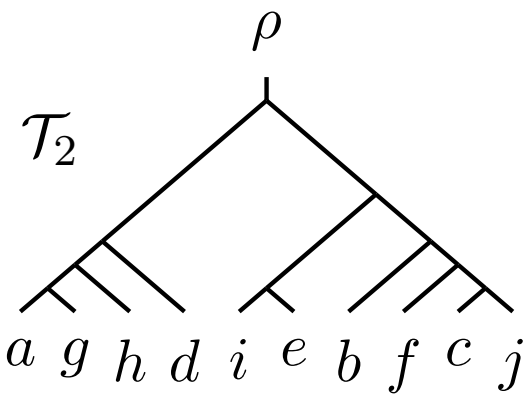

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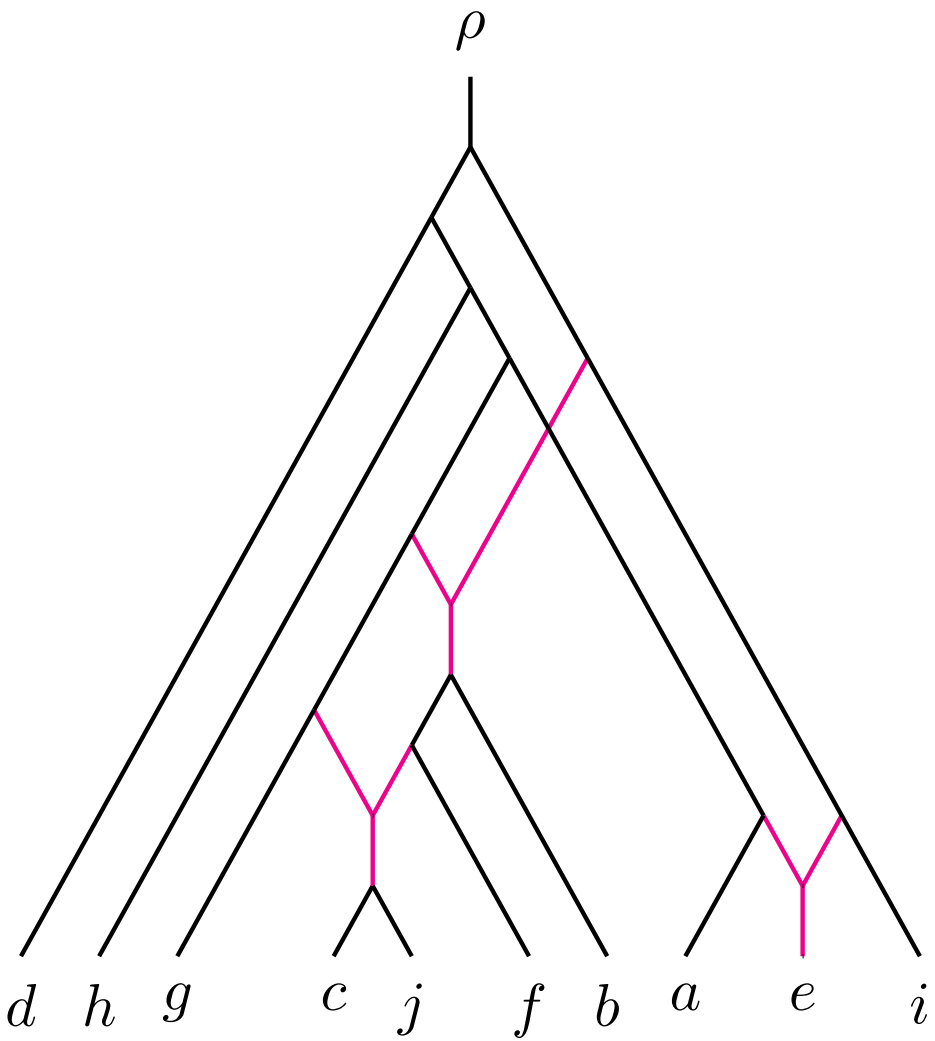

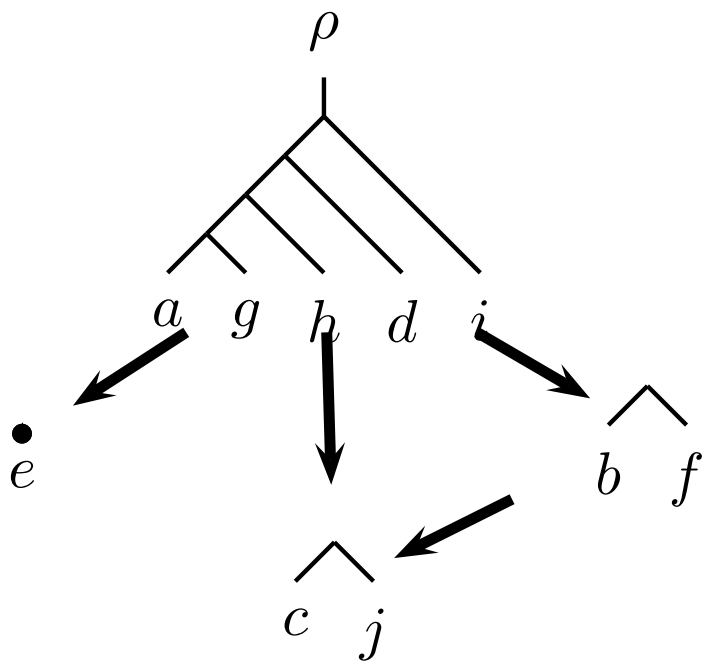

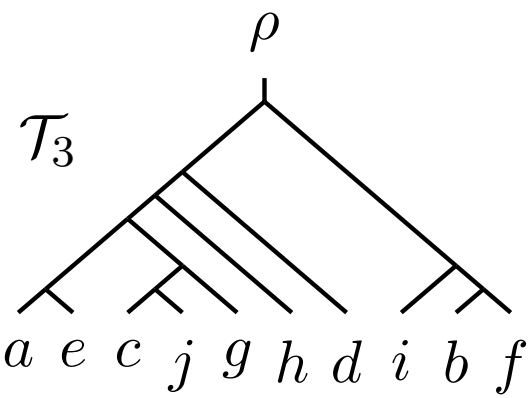

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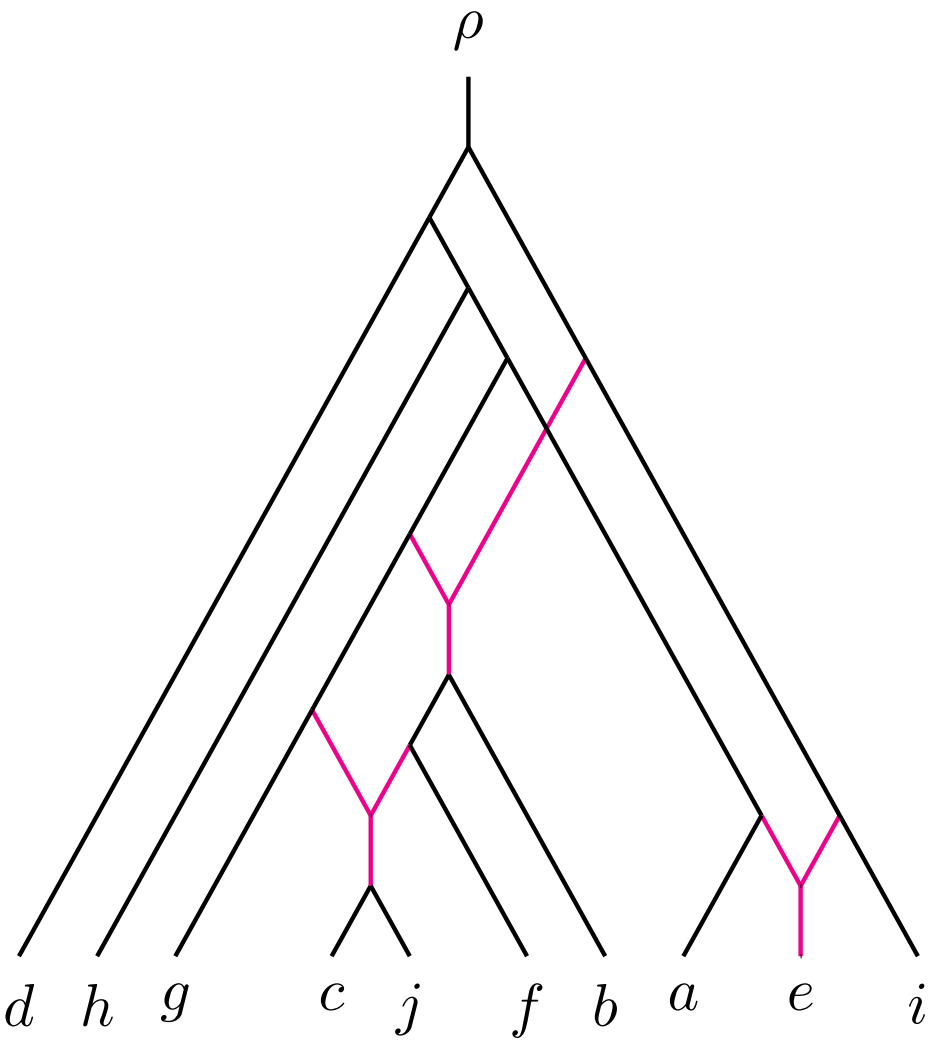

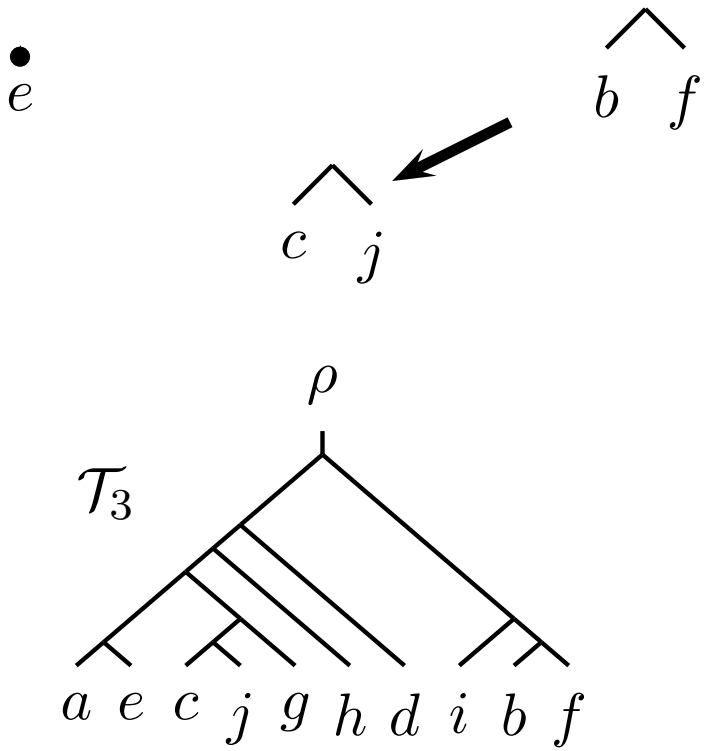

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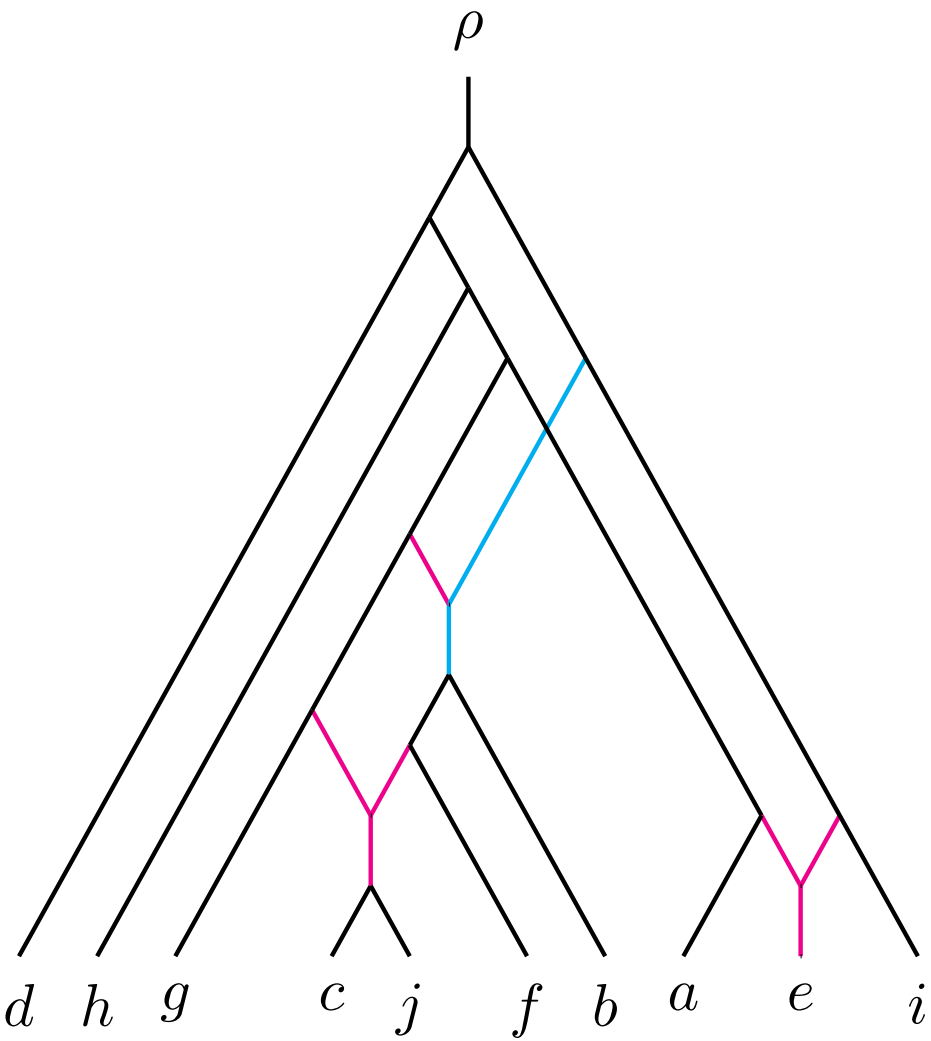

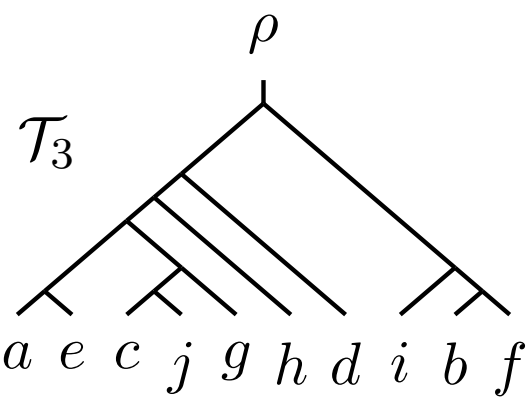

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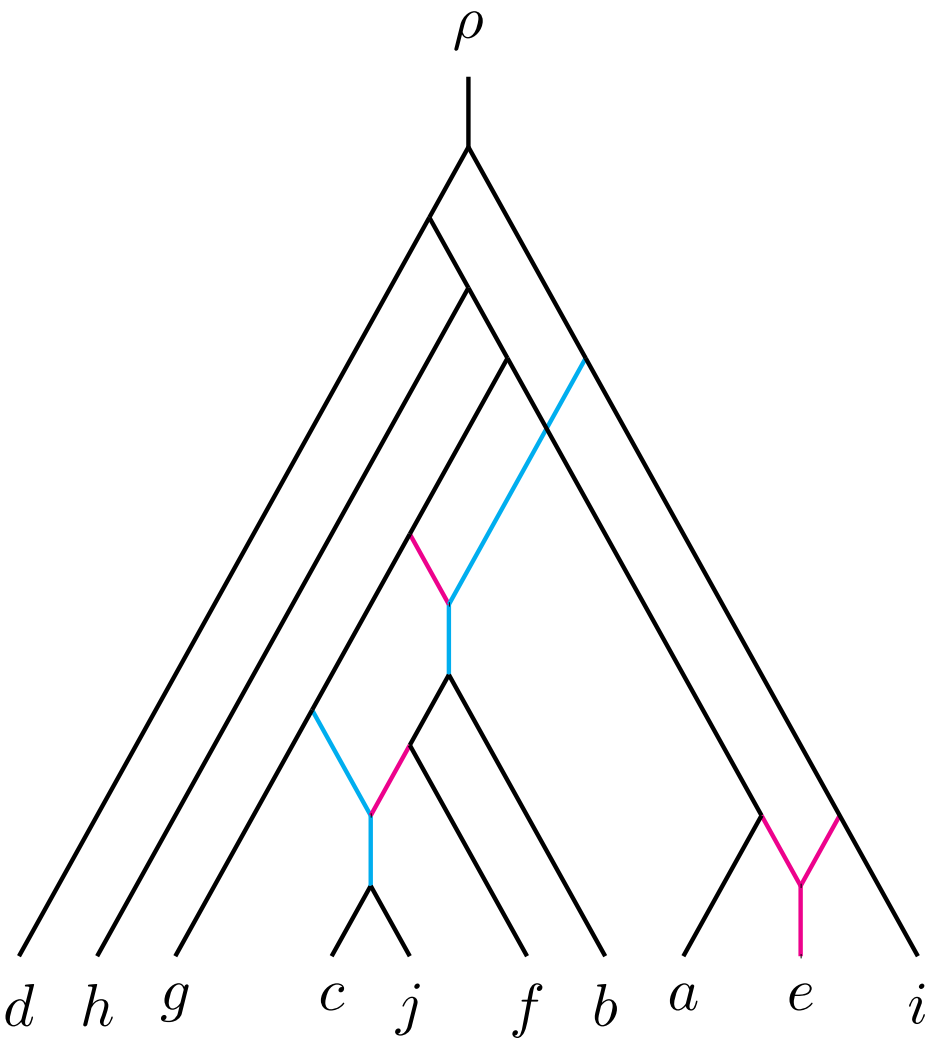

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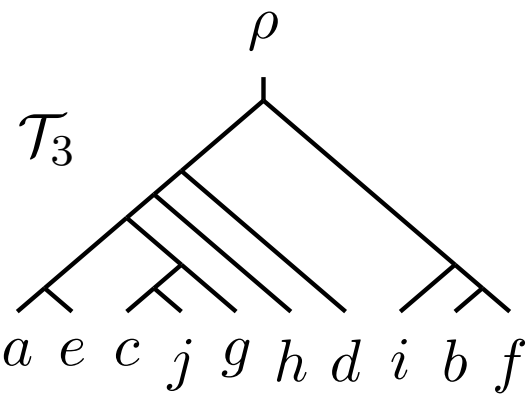

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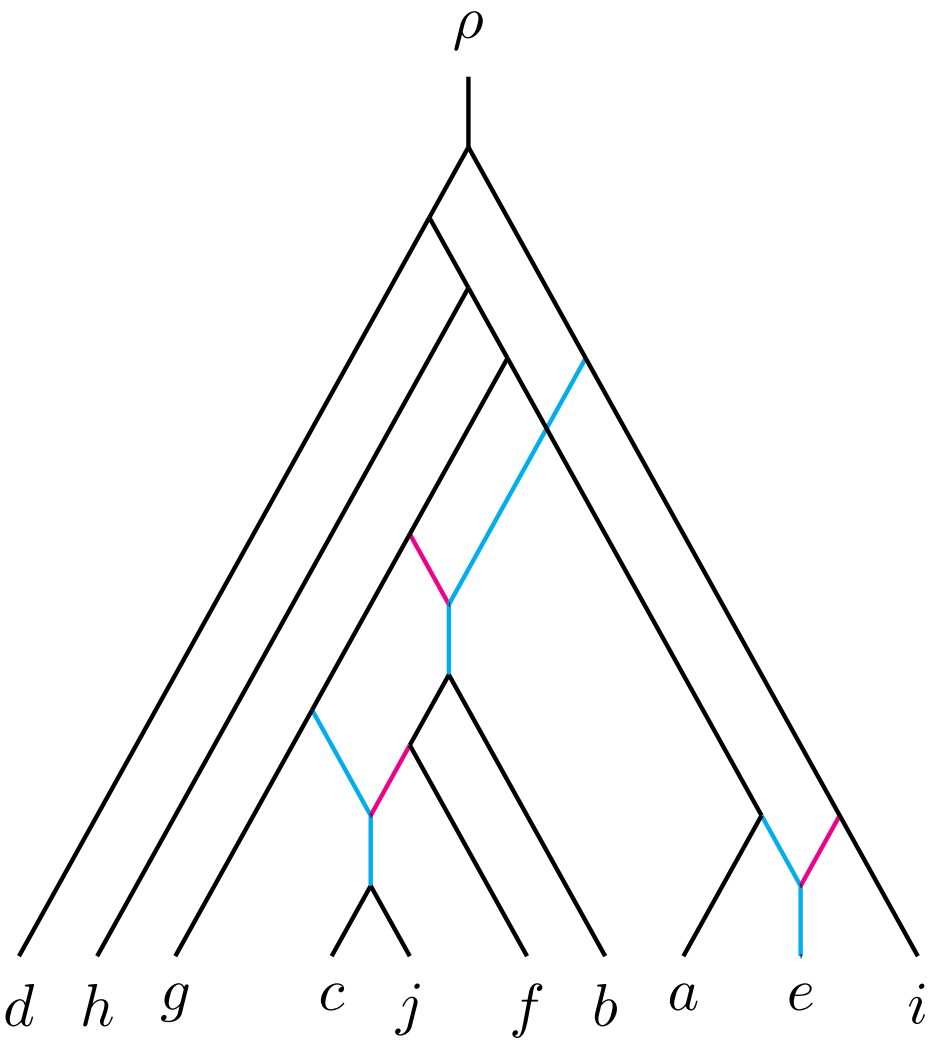

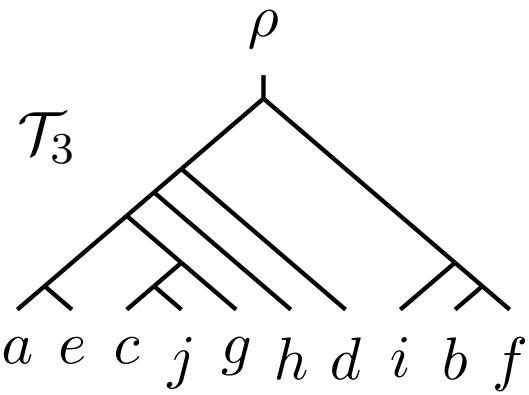

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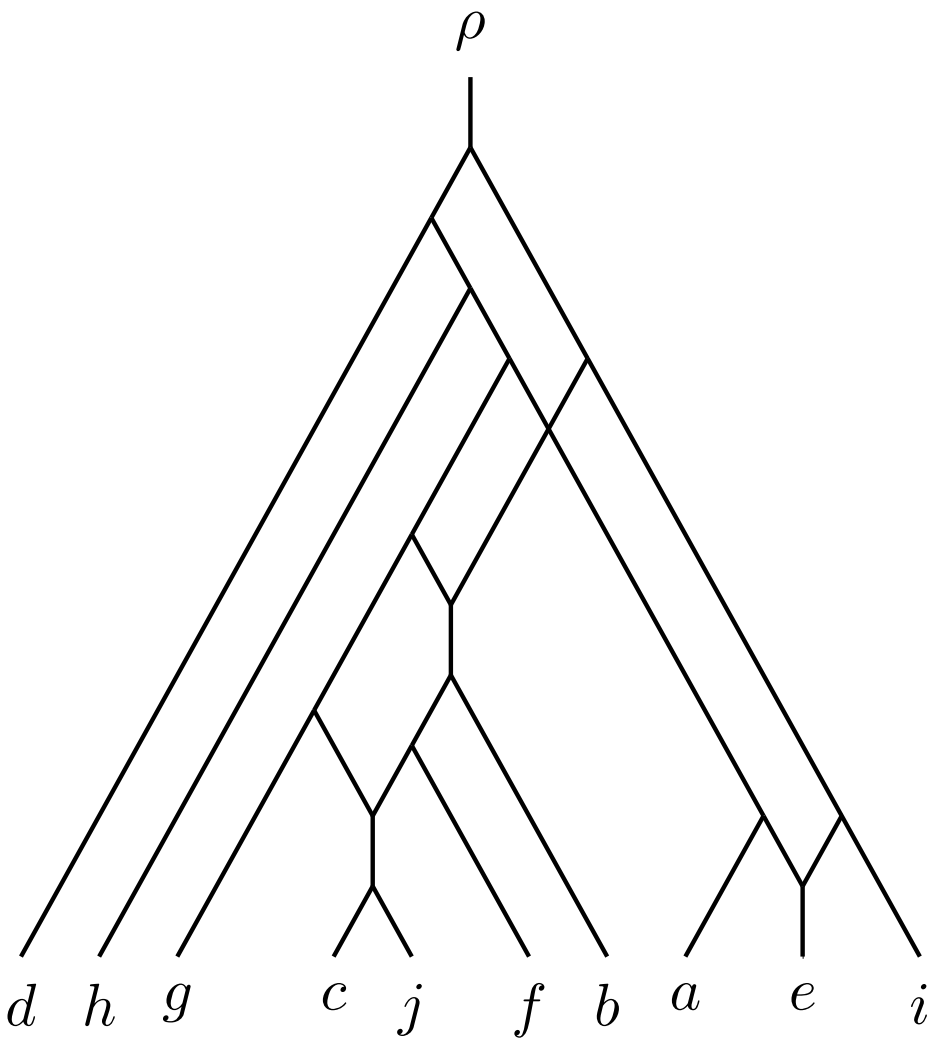

Great, so what's the problem?

$$\mathscr{T} = \left\{ \begin{array}{ccccc} \rho & & \rho & & \rho \\ \text{tree 1} & , & \text{tree 2} & , & \text{tree 3} \\ a\ b\ e\ c\ d & & a\ b\ c\ e\ d & & a\ c\ d\ b\ e \end{array} \right\}$$

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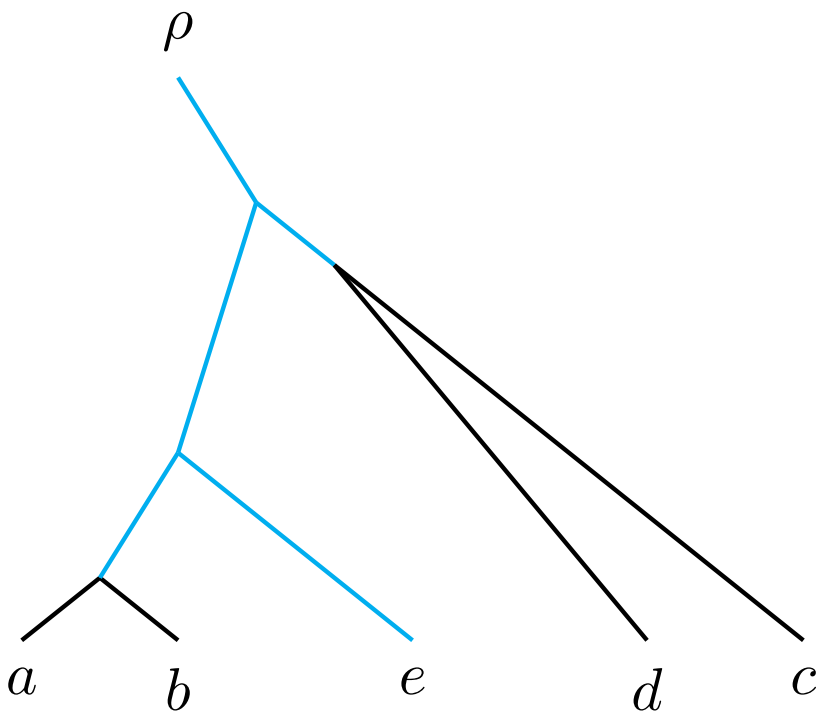

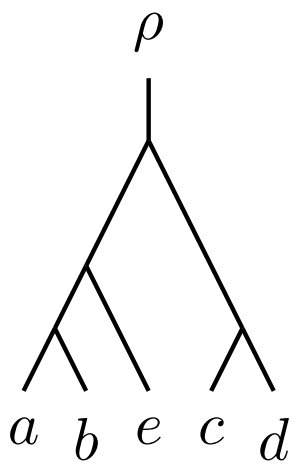

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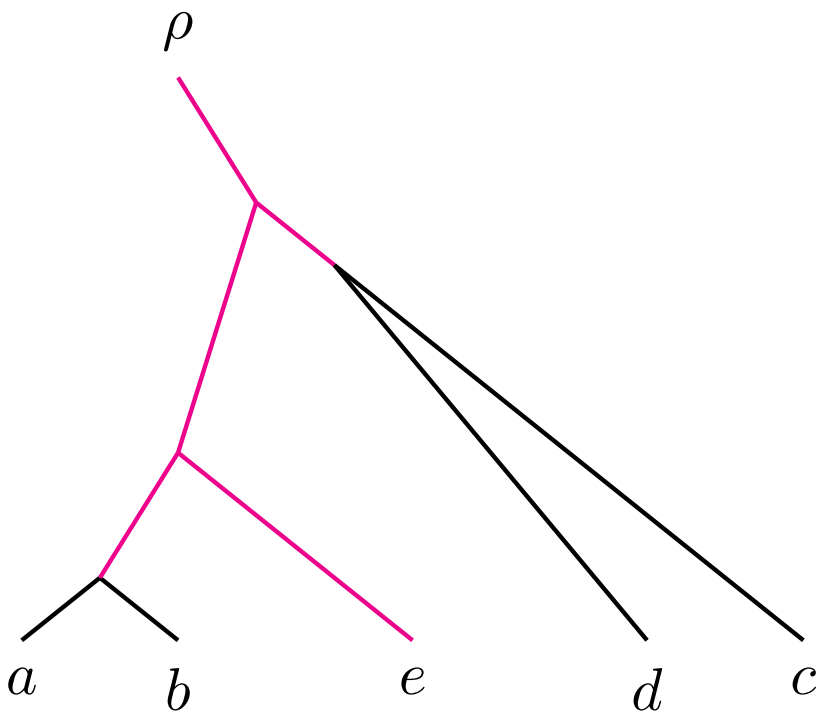

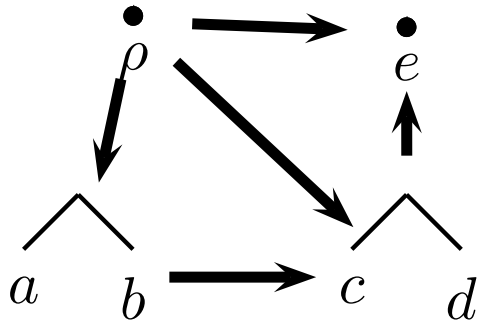

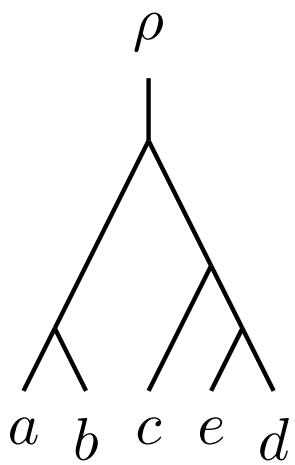

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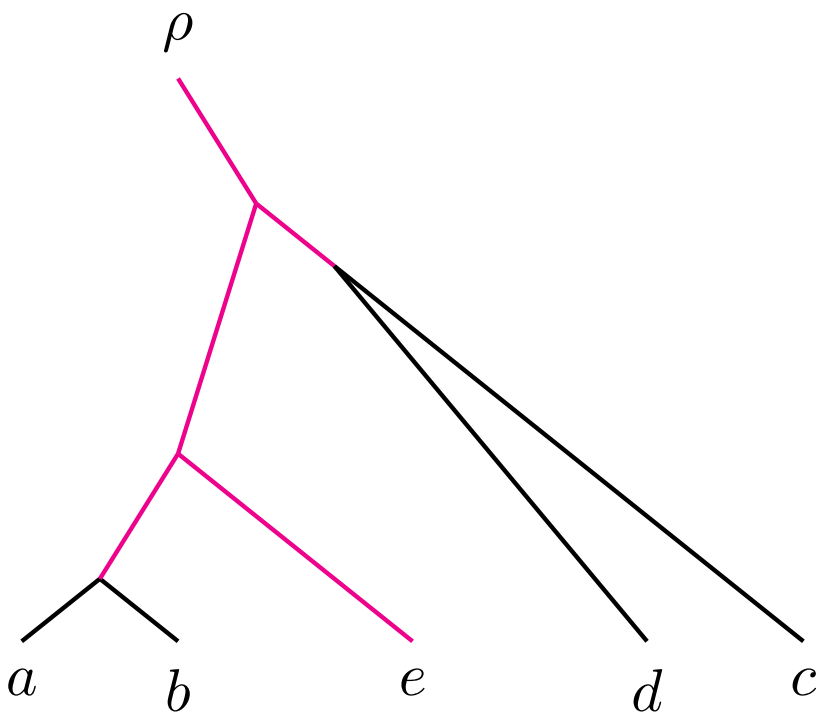

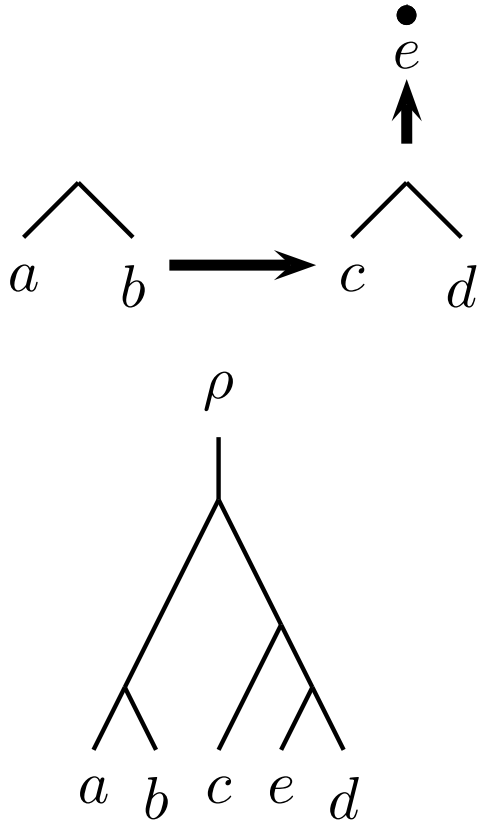

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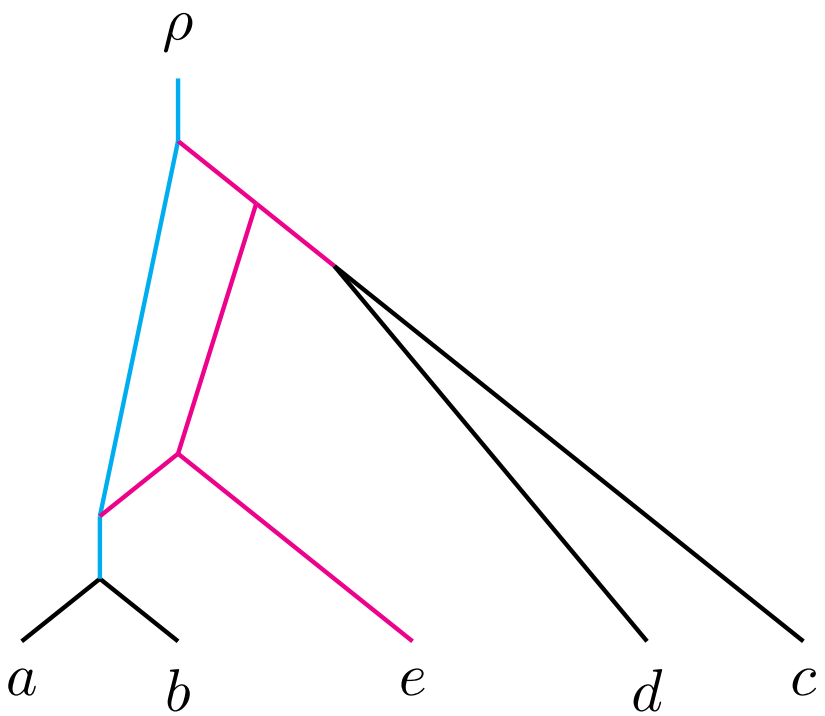

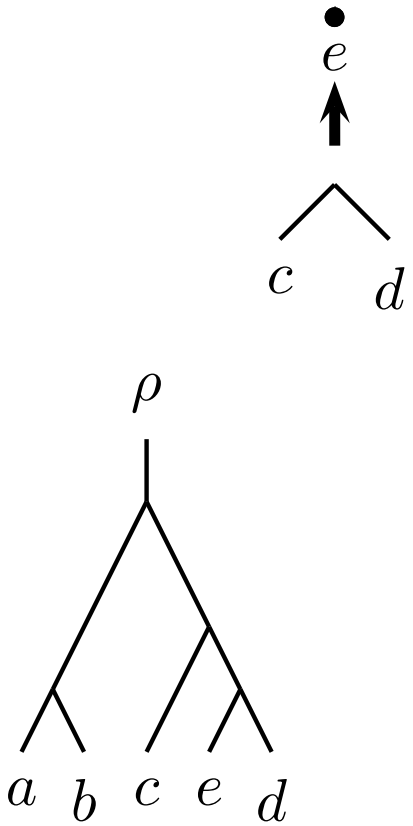

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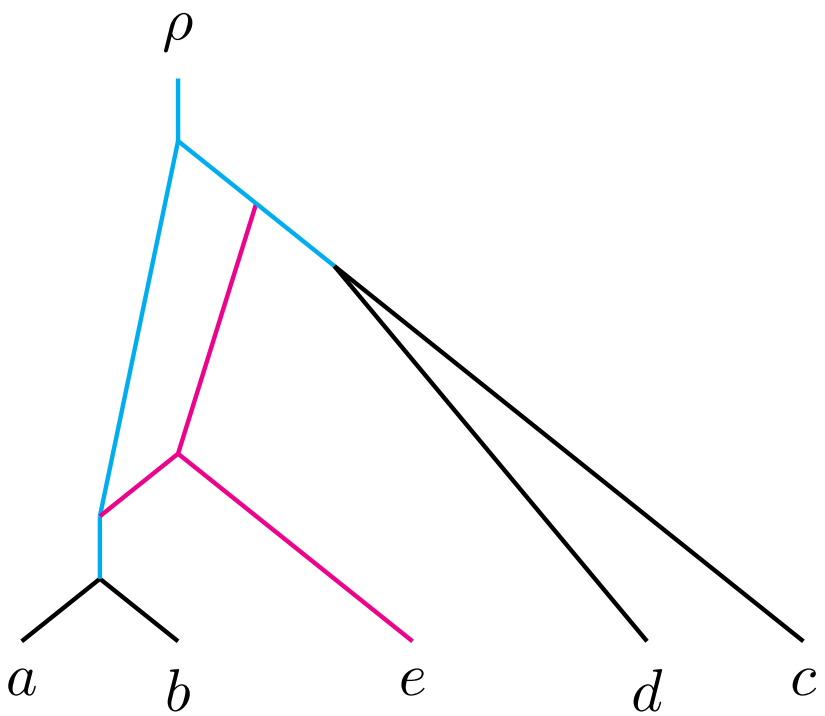

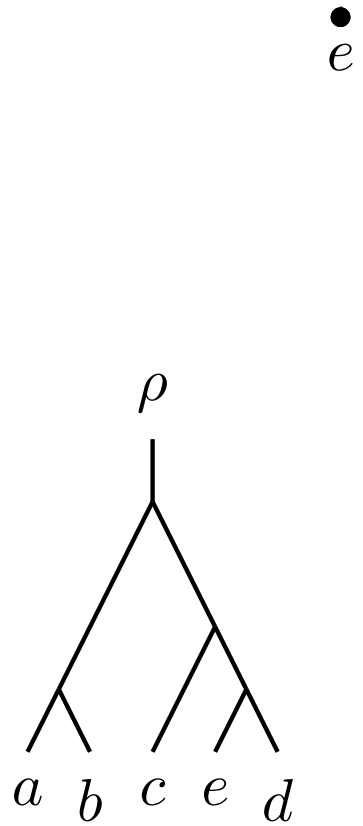

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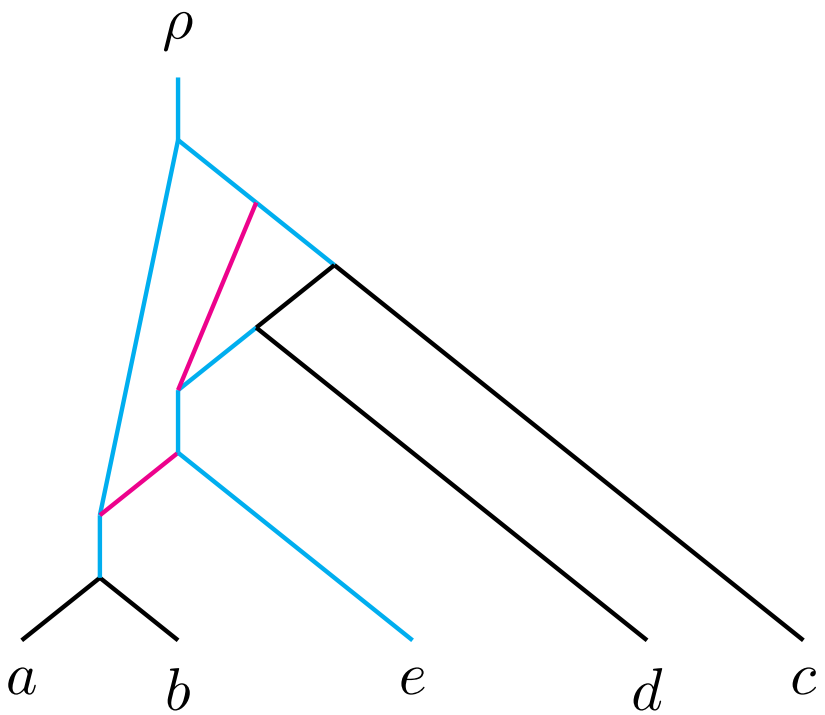

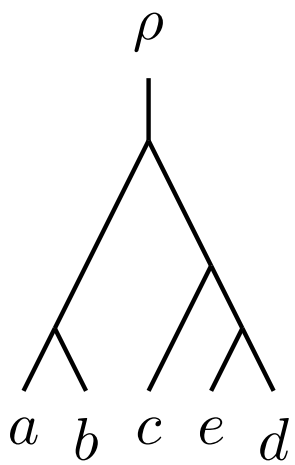

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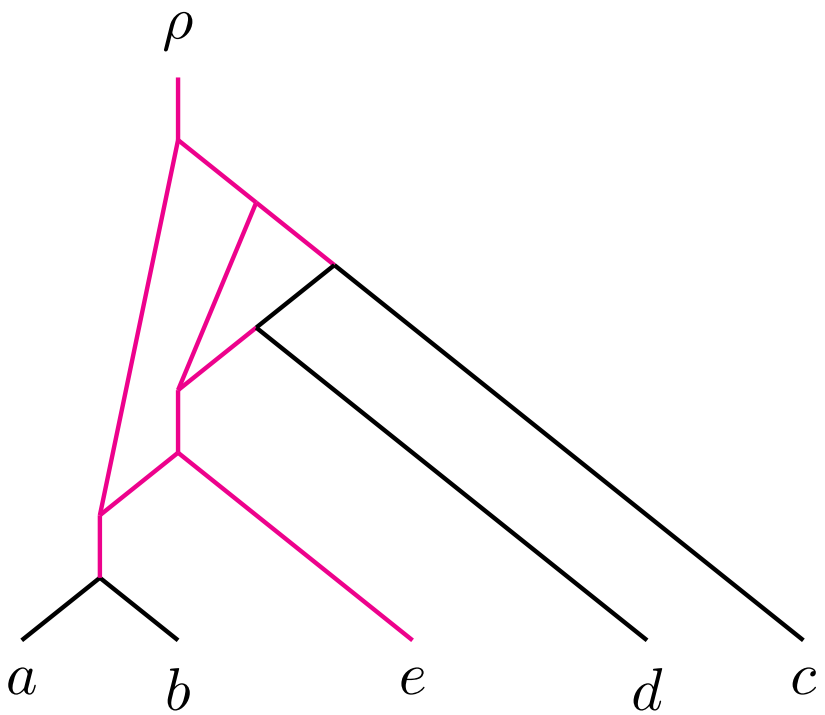

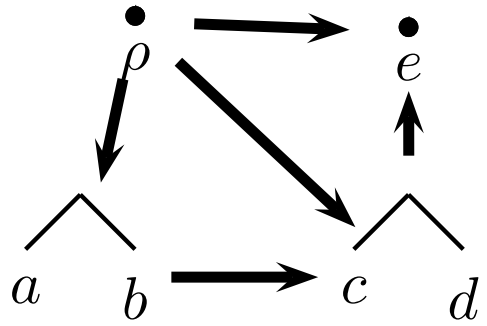

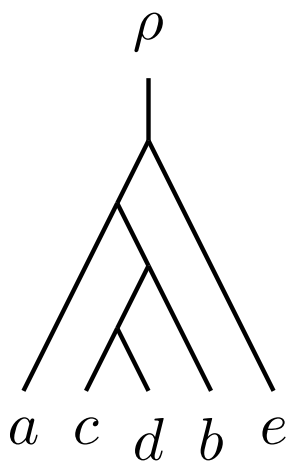

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- Can *still* get into positions where it's not possible to add a required arc without creating a cycle.

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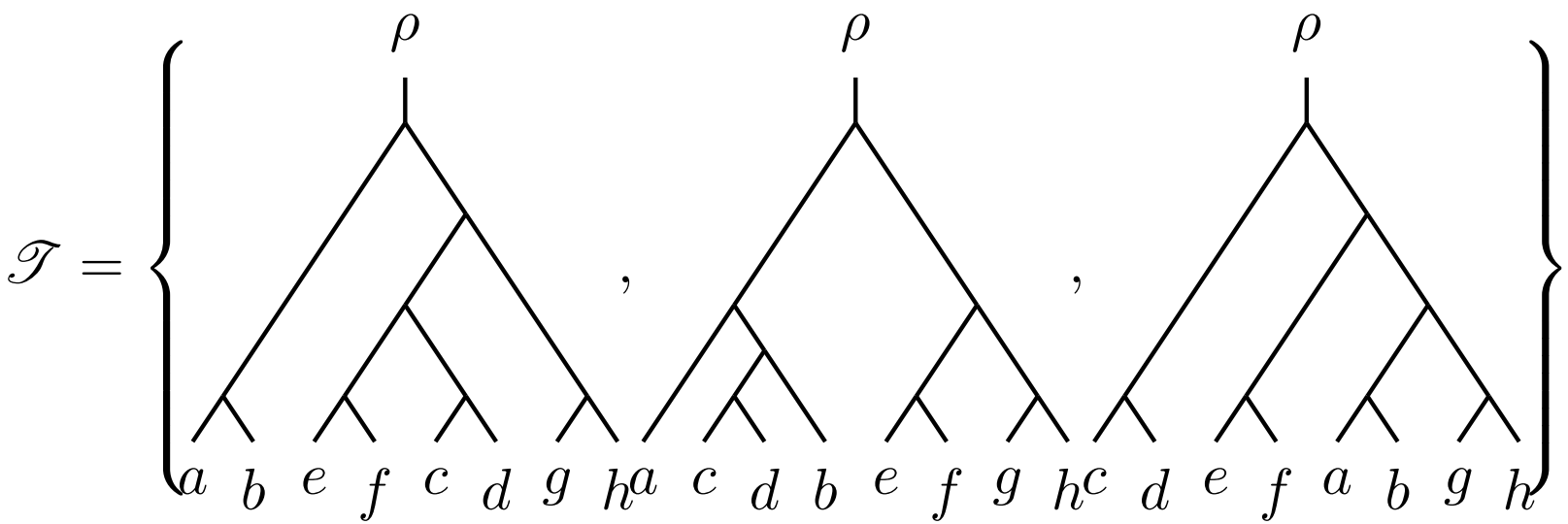

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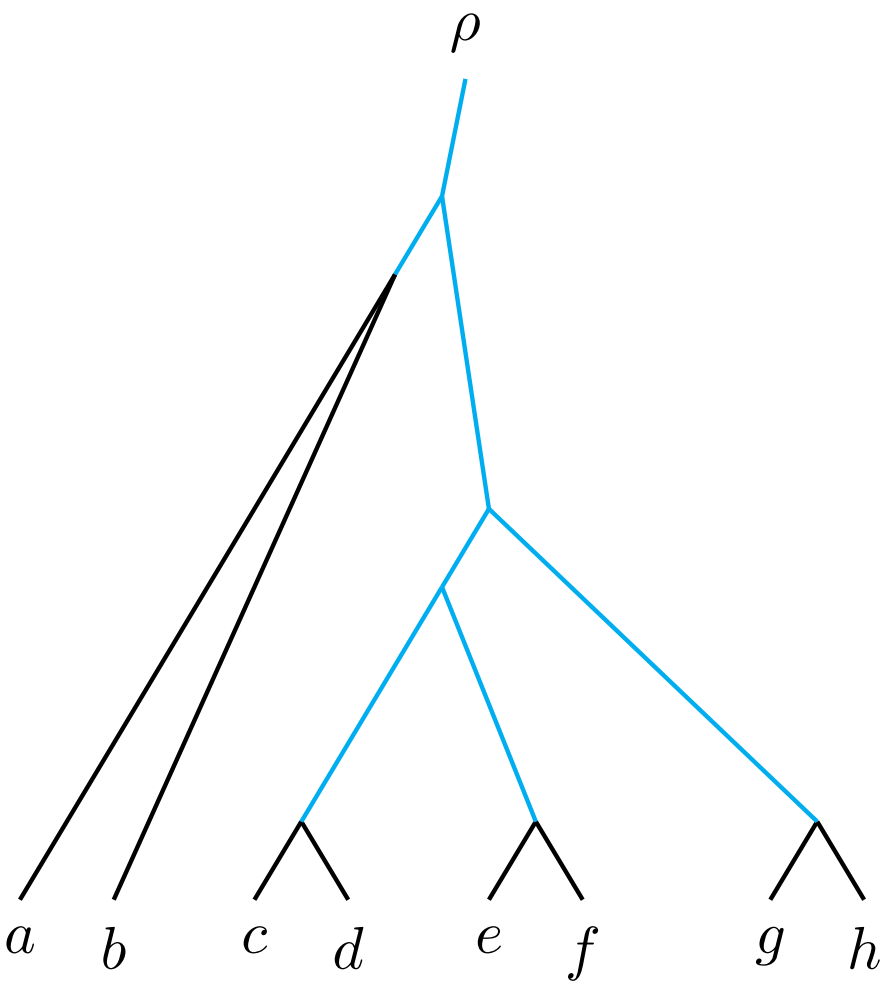

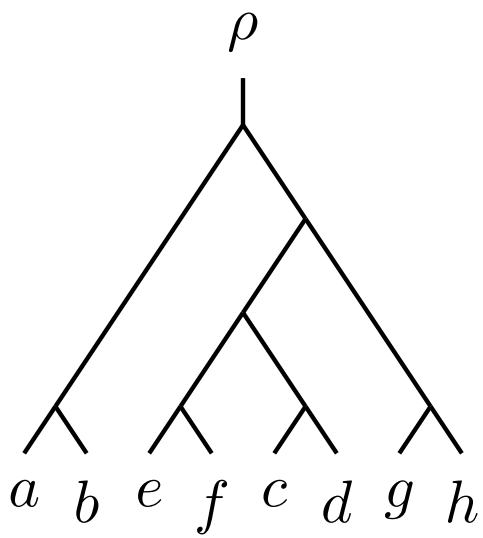

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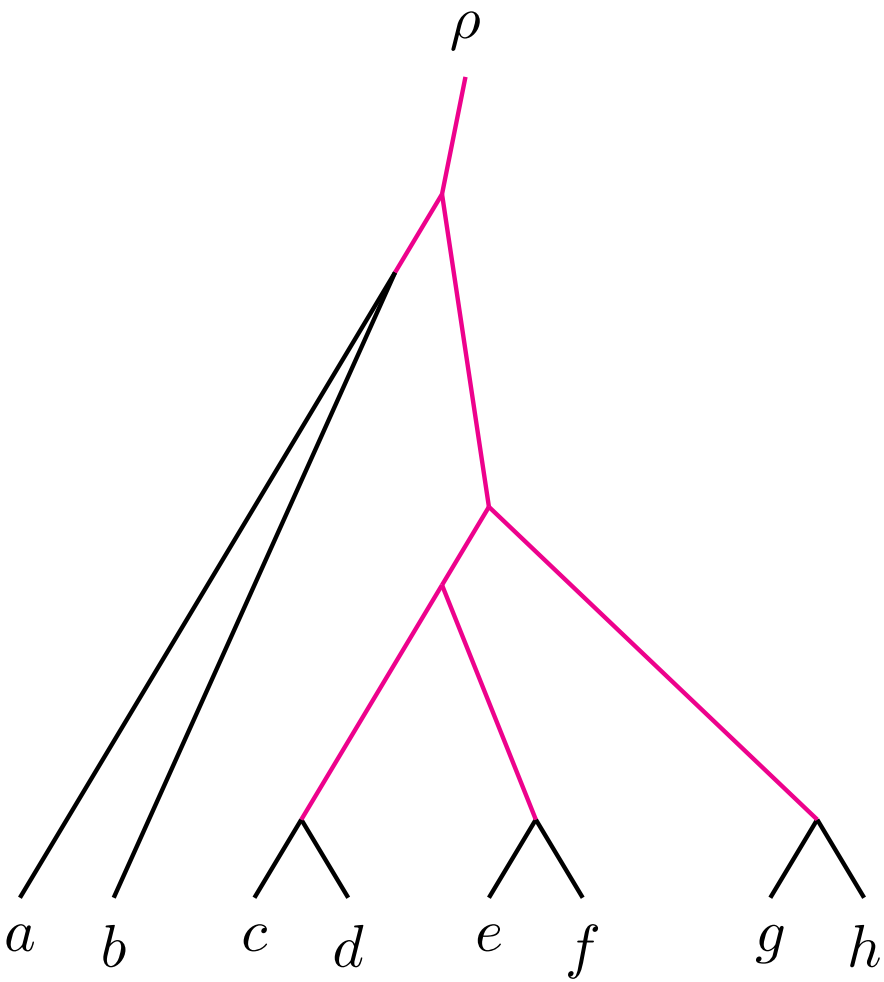

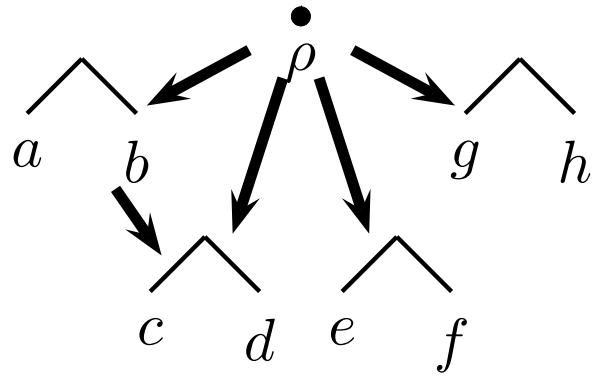

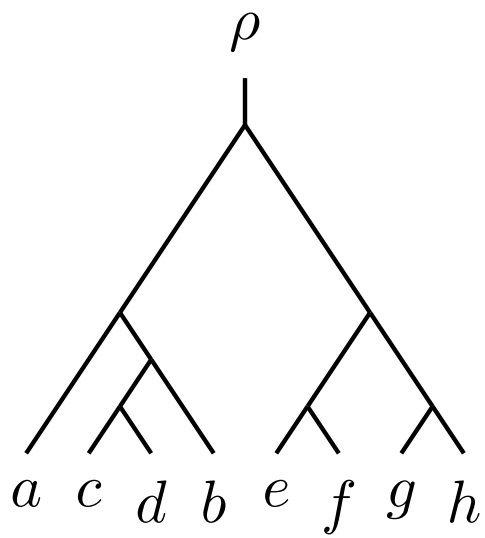

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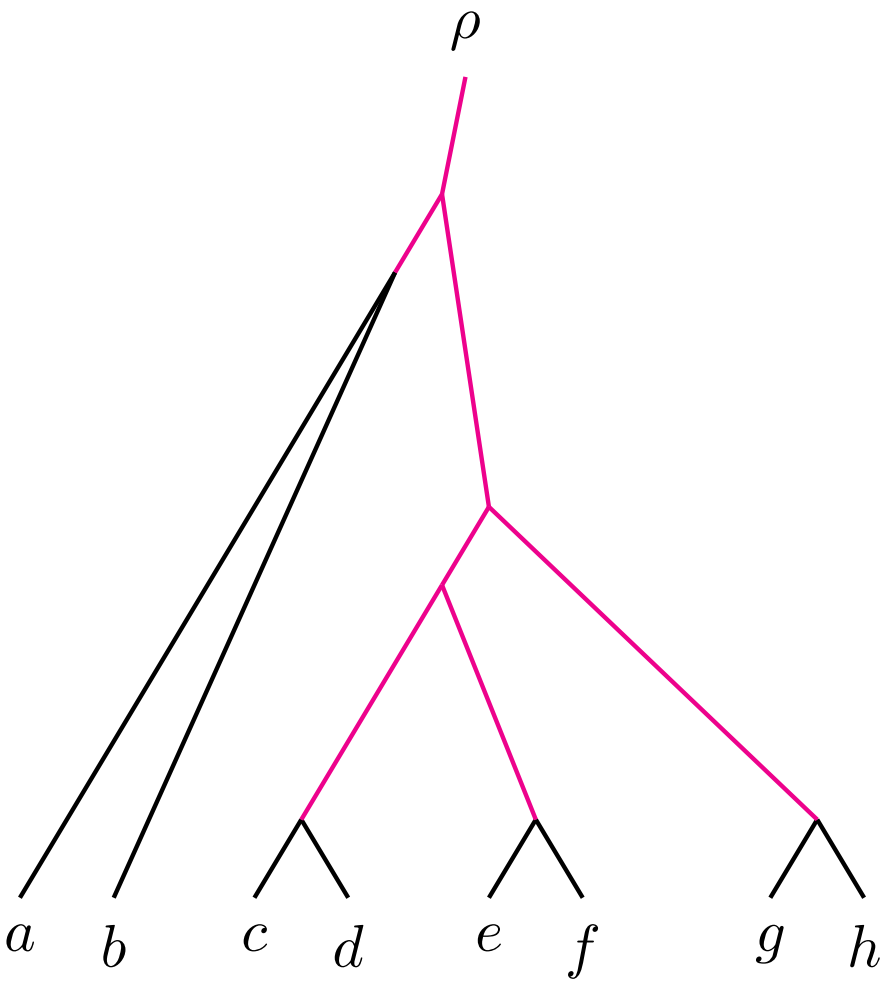

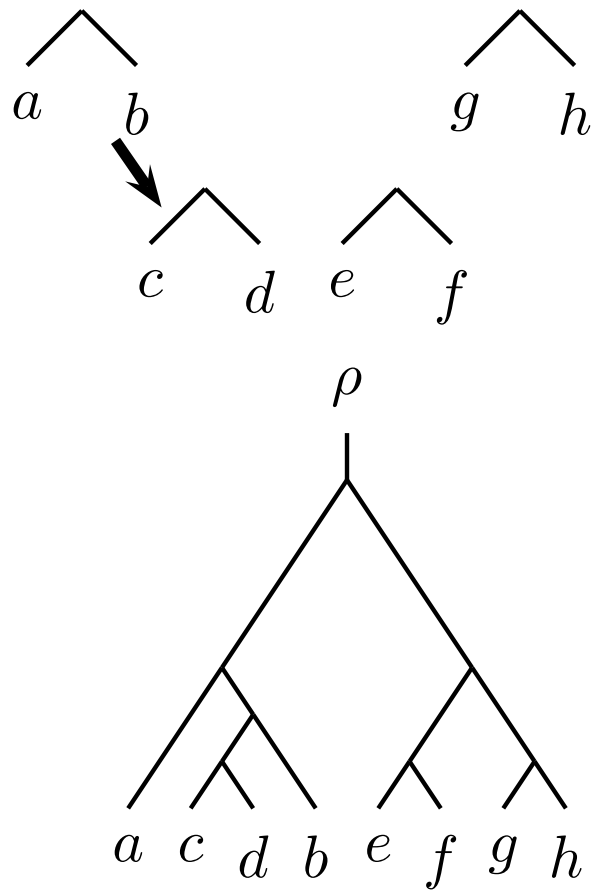

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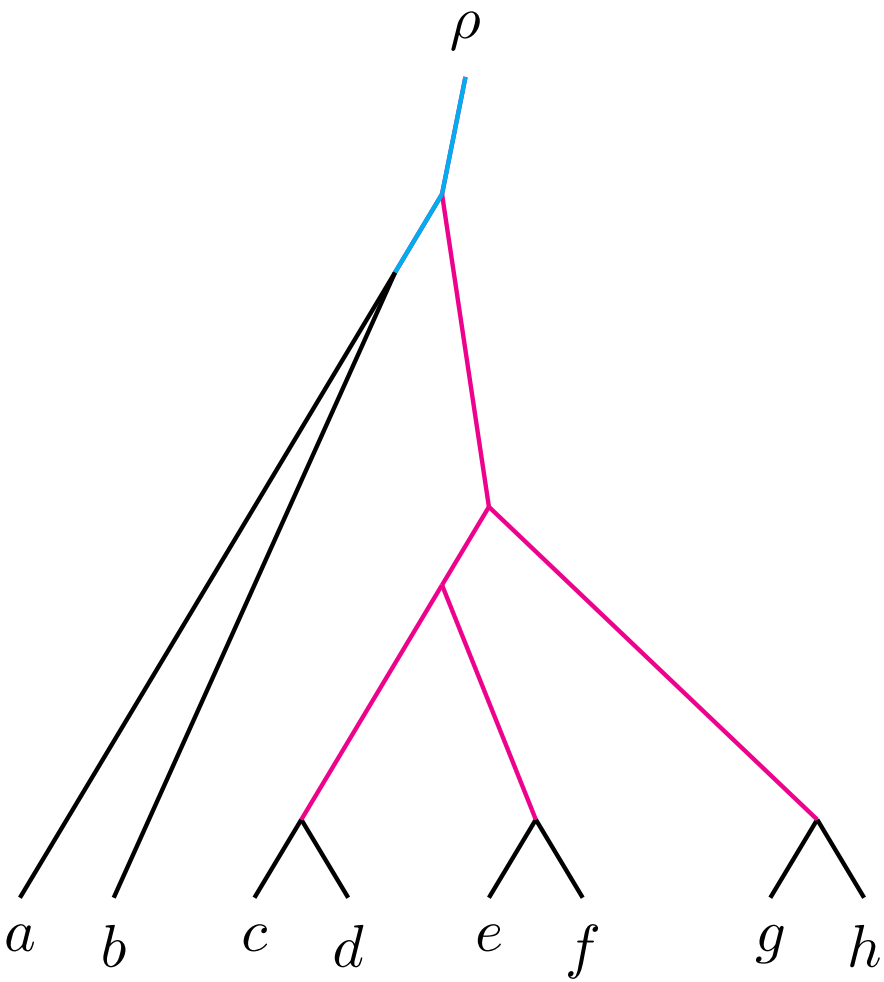

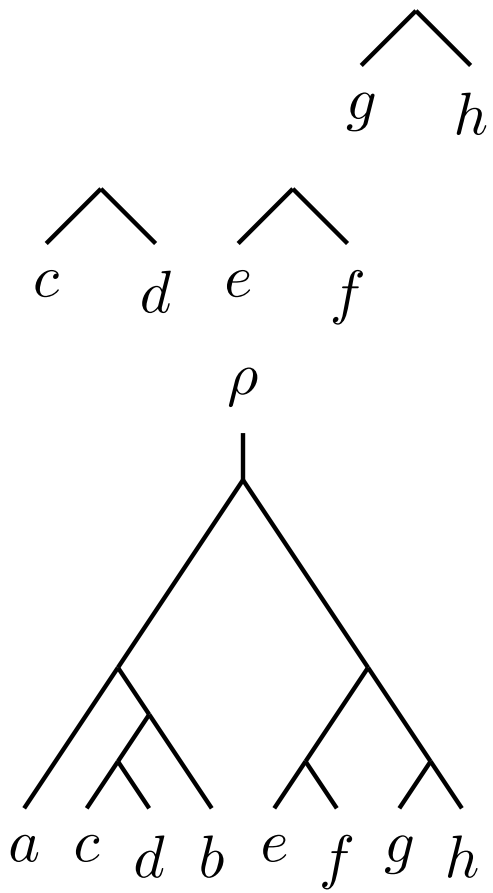

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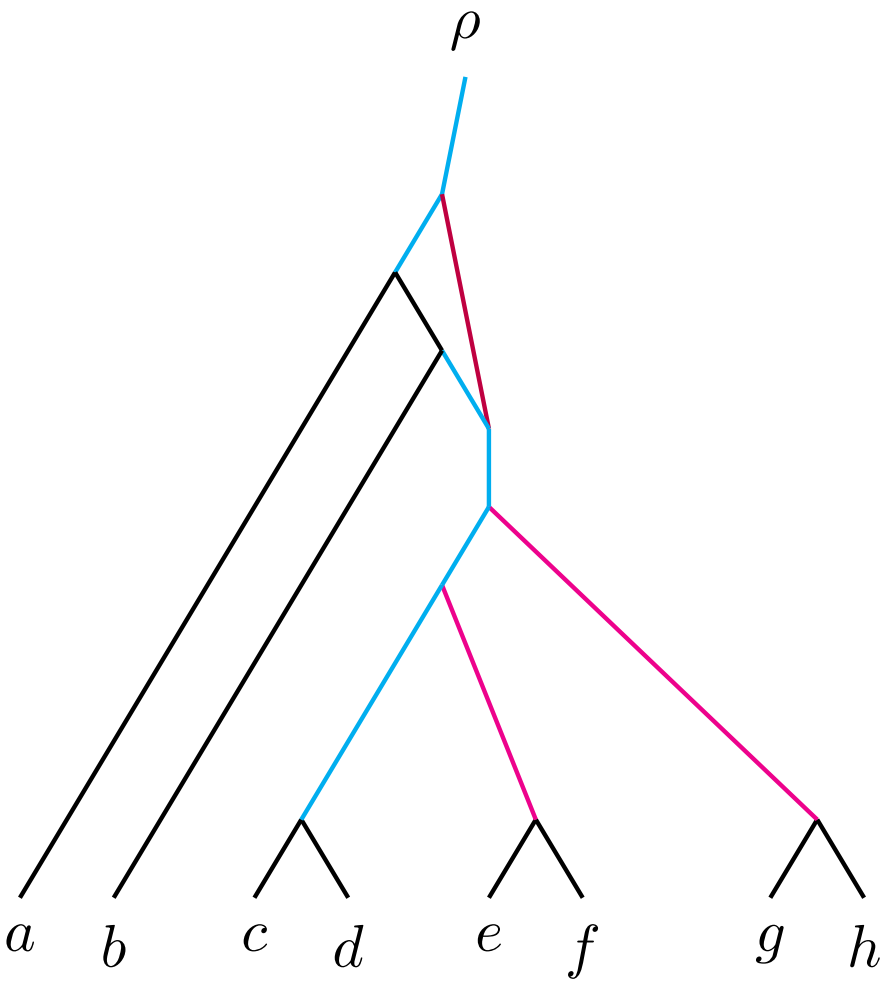

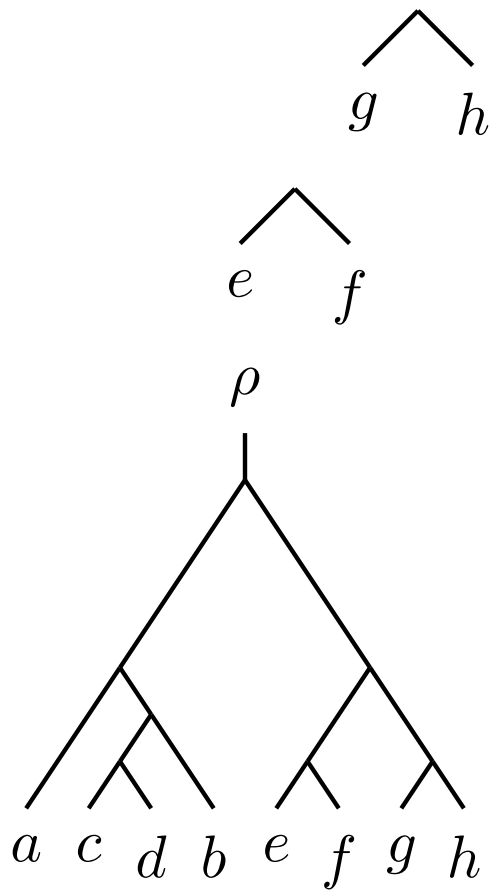

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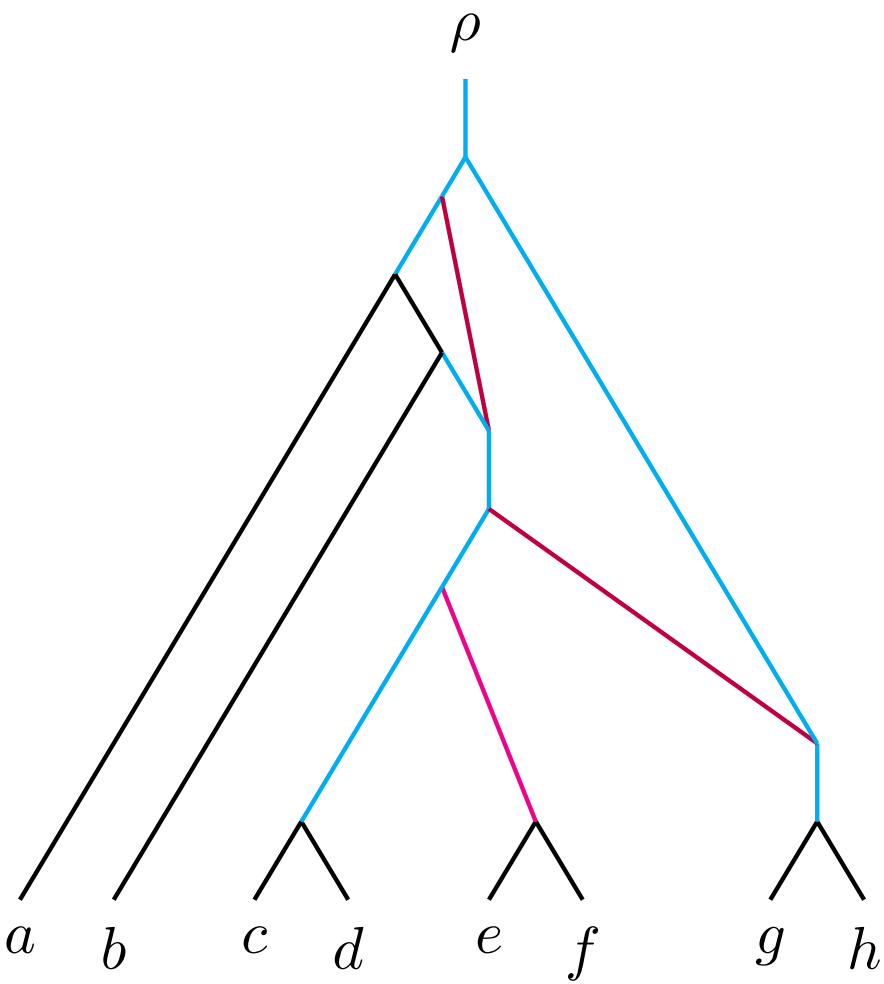

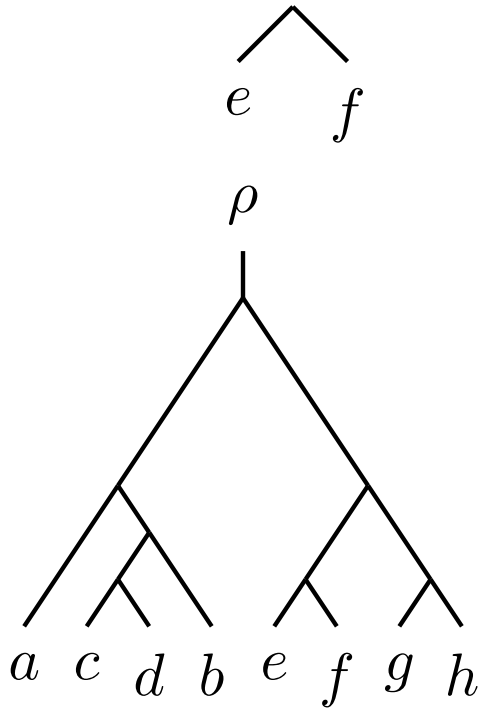

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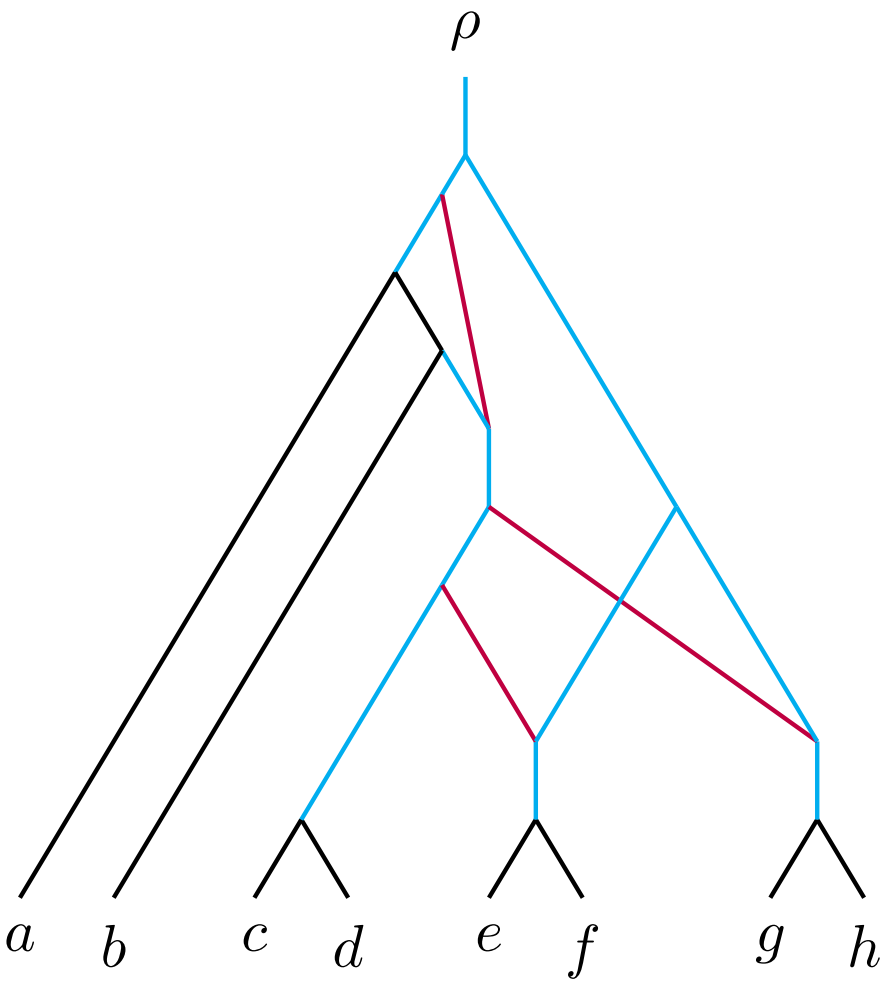

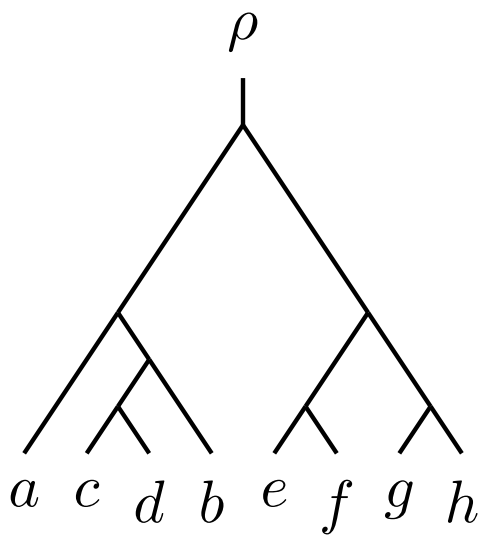

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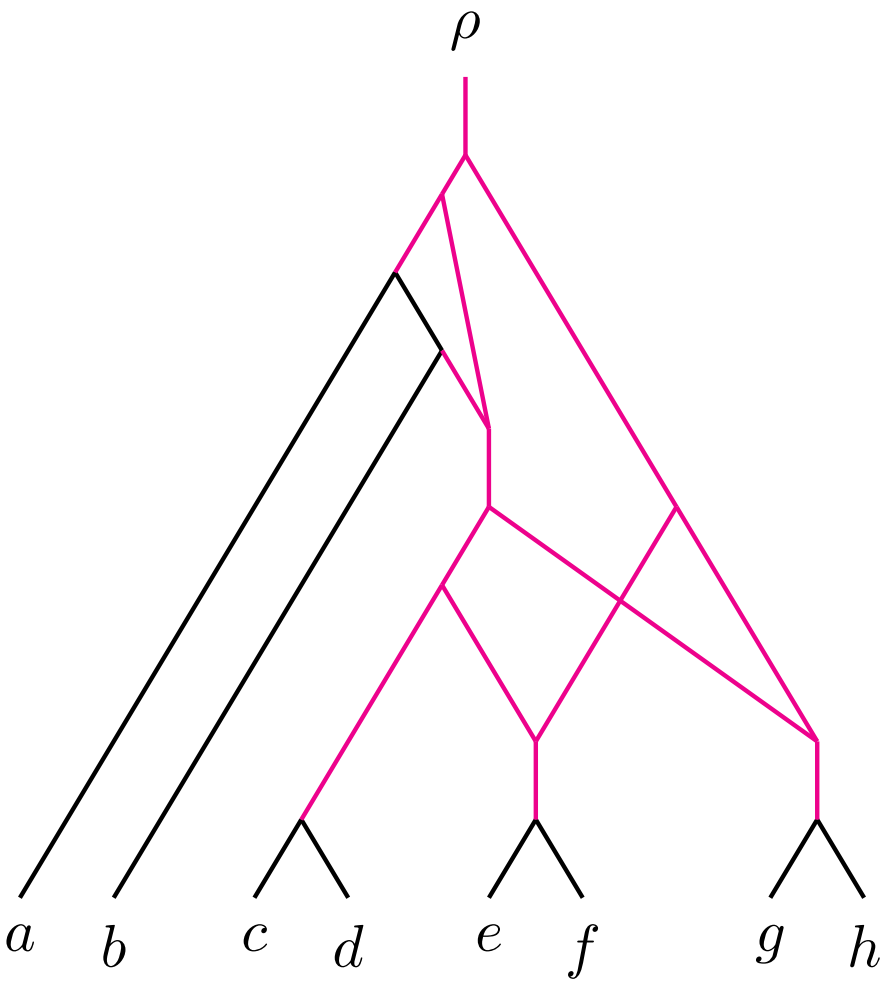

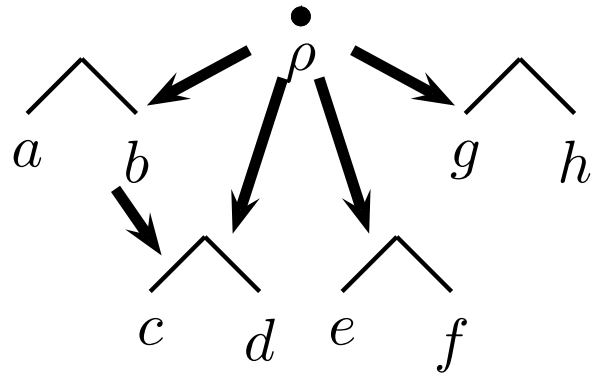

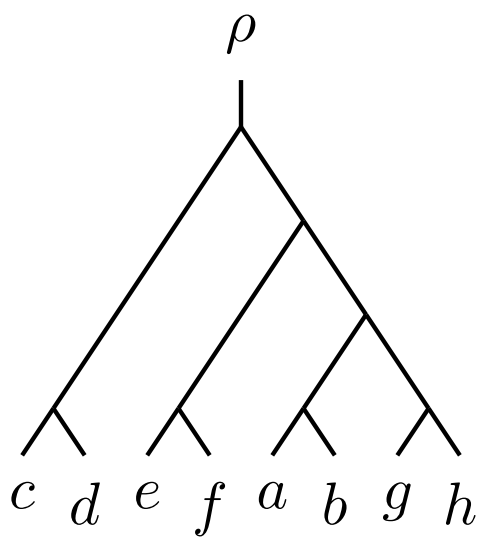

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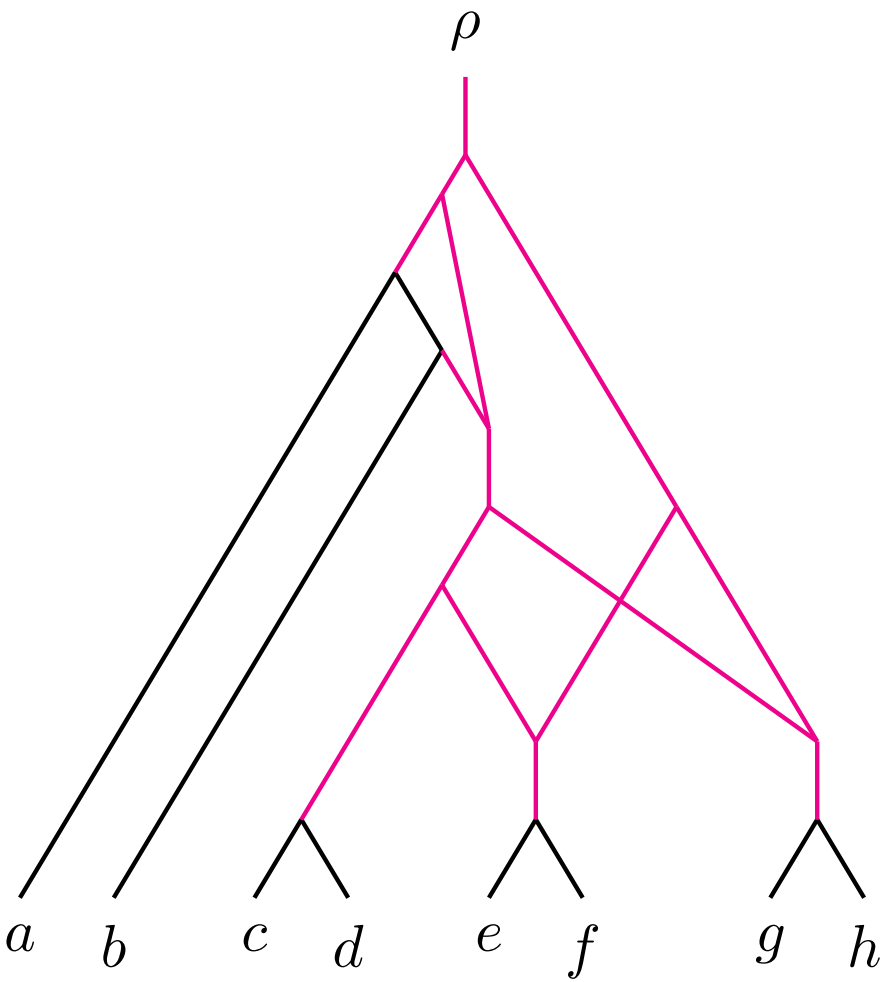

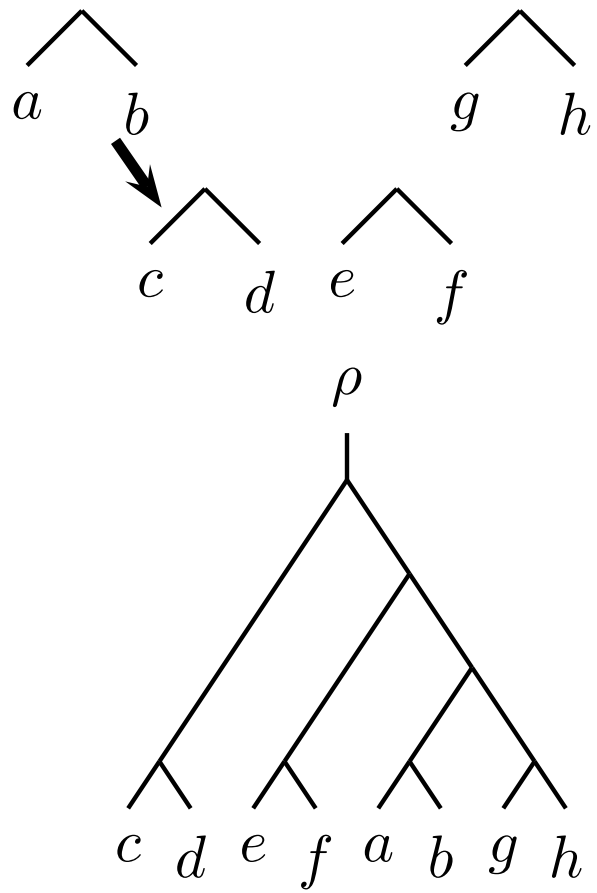

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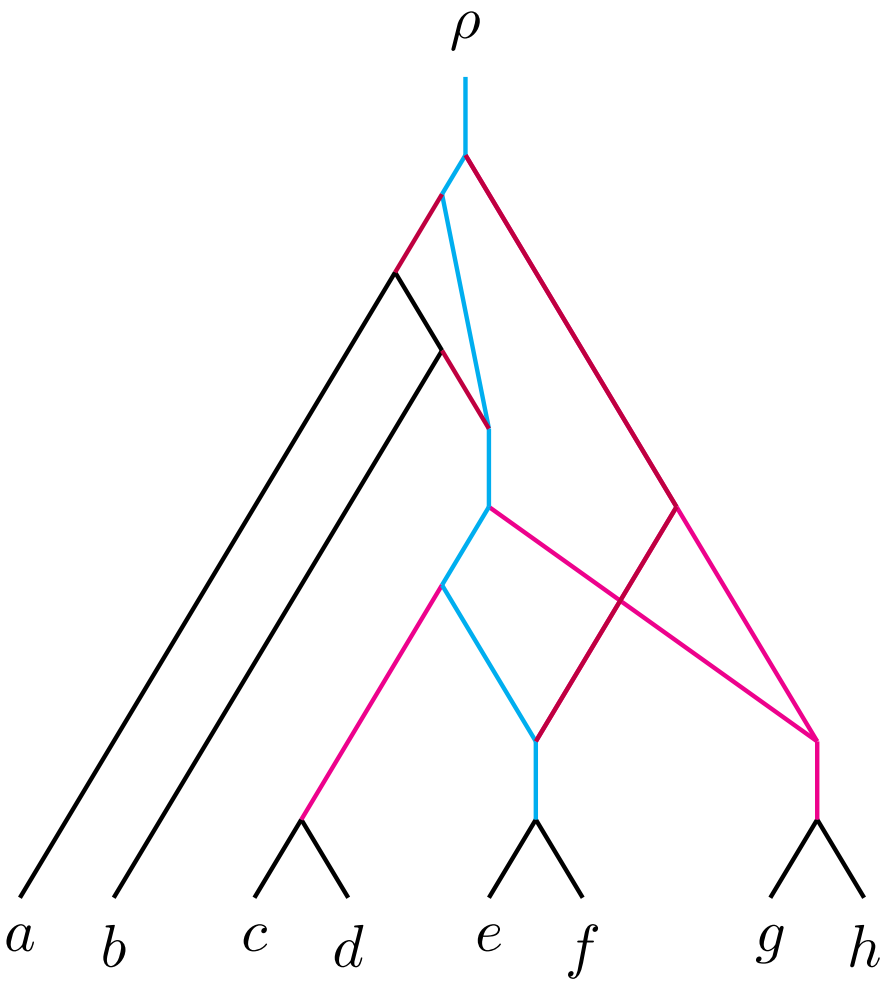

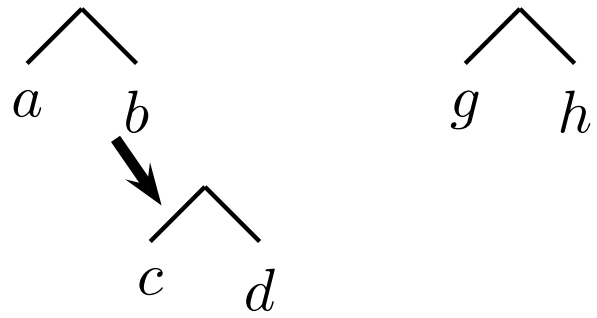

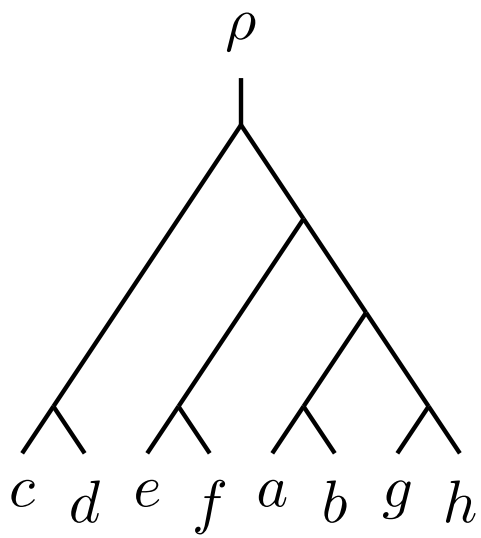

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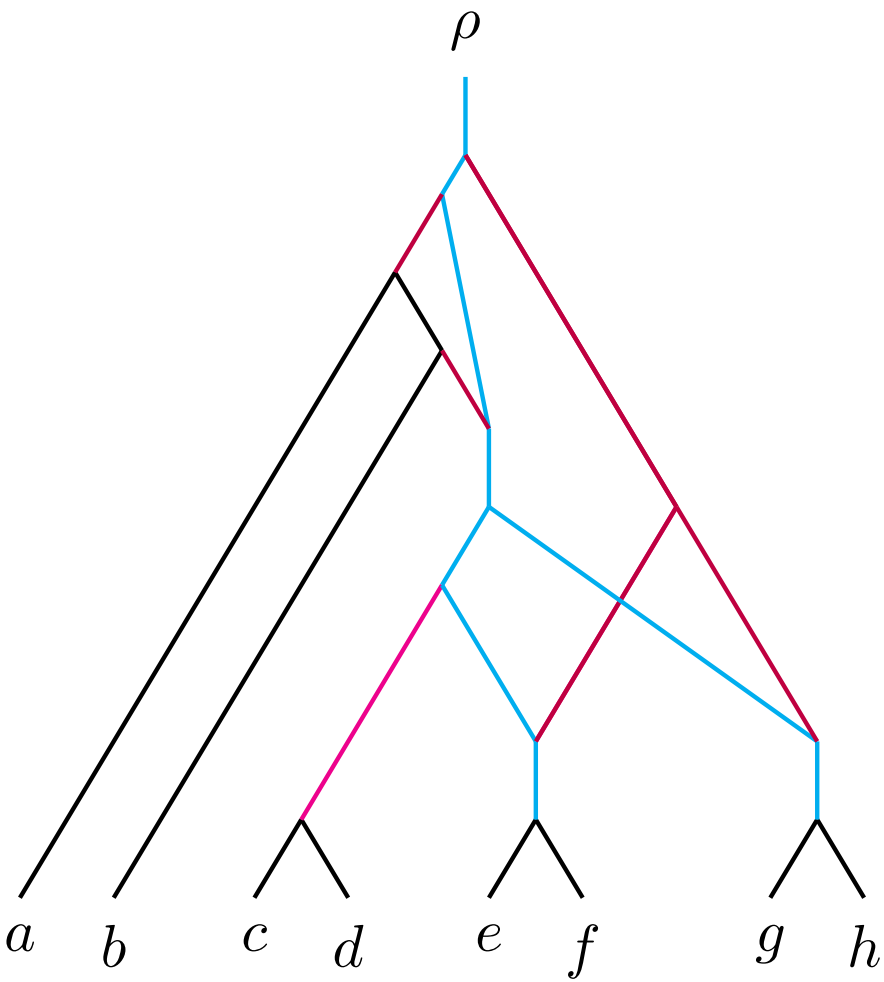

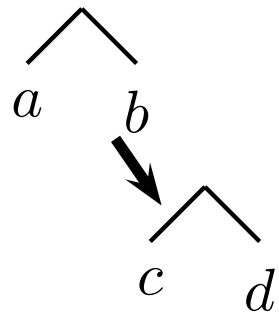

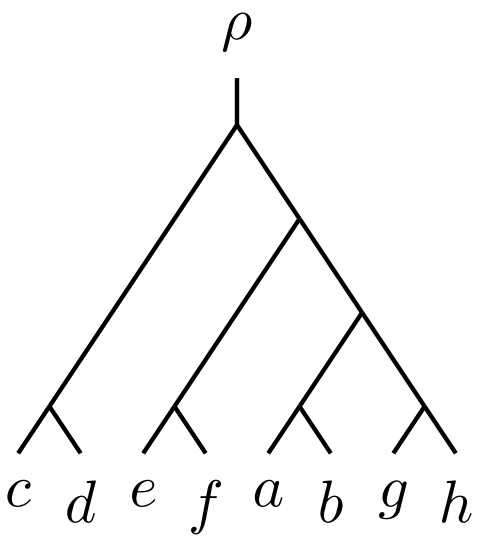

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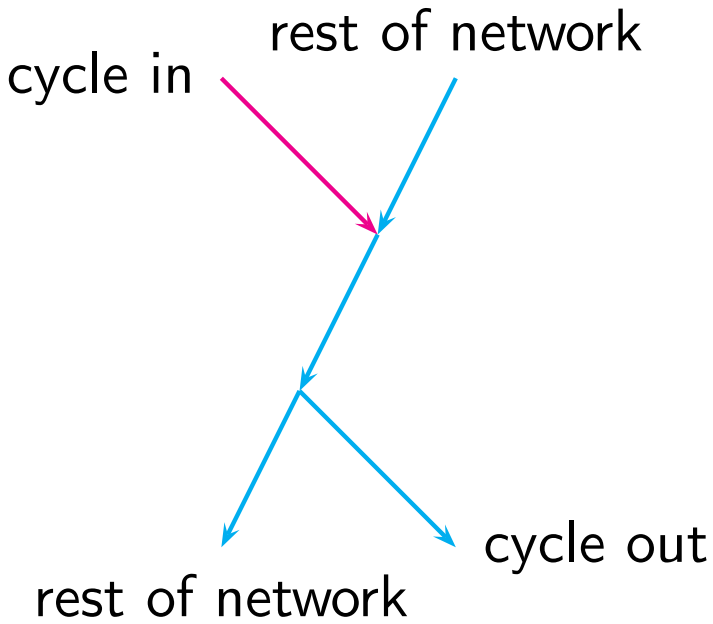

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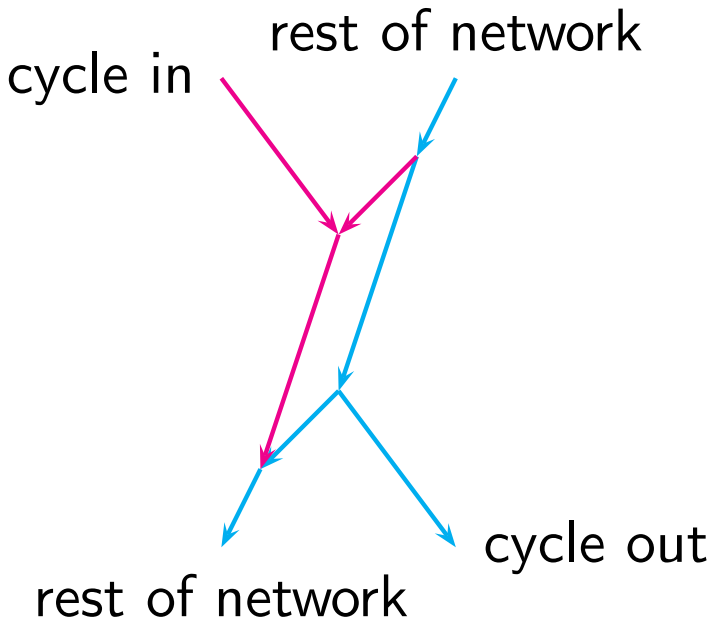

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Other possible uses:

- Enumerate through all networks that display $\mathscr{T}$ and induce a forest that is a superforest of a provided $\mathcal{F}$
- Thus give a fairly brute force algorithm to calculate the hybridisation number of sets of trees of arbitrary size.

Open Questions

Question

In the two tree case the size of the maximum acyclic agreement forest is one greater than the hybridisation number of the set of trees. If given more than two trees is there any similar relation between the networks of smallest hybridisation number that displays the trees and maximum acyclic agreement forests? If not is there a relation between it and the maximal acyclic agreement forests?

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