Markovian decision-support model for patient-to-ward assignment problem in a random environment

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Patient to Ward Assignment Problem

- A generalisation of the advanced patient scheduling problem (APAS).
- Patients are allocated to beds in different wards.
- We take into account the patients' needs, their priorities and the availability of resources.
- We construct a Markov Decision Problem (MDP), in which:
  - Also, decisions are made at the time of each time period of a fixed length,
  - Allocation decisions involved several arrived patients (of different type).
- Our goal is to find the optimal assignment of patients to wards at a minimum expected cost.

We use the following notations for our math model:

- $\mathcal{I} = \{1, 2, \ldots, I\}$: the set of all patient types and contains all required information,
- $\mathcal{K}$: the set of ward in the hospital,
- $\mathcal{T}$: the planning period,
- $\mathcal{B}_{\mathcal{K}}$: the number of beds in ward $k \in \mathcal{K}$,
- λ_i : the arrival rate of type-i patient,
- $Q_i(t)$: the random variable recording the number of arrivals of type-i patient during time t, and $Q_i(t) \sim Poi(\lambda_i t)$,
- q_i : the number of newly arrived type-i patients.

Random Departure

- Length of Stay (Latouche & Ramaswami, 1999): $LoS_{k,i} \sim PH(\gamma^{(k,i)}, \mathbf{P}^{(k,i)})$,
- Random departures during interval t: $Z_{k,i}(t) \sim Bin(n_{k,i}, p_{k,i}(t))$,

$$p_{k,i}(t) = Pr(LOS_{k,i} \leq t) = 1 - \gamma^{(k,i)} \left(\mathbf{P}^{(k,i)}\right)^t \mathbf{1} \; \forall t \in \mathcal{T}, \quad (1)$$

- where $\mathbf{1}$ is a column vector of ones of appropriate size.

- State space:
  - The set of state space is denoted by $\mathcal{S}$,
  - The information of the system at time t is given by $s_t \in \mathcal{S}$, defined as $s_t = ([n_{k,i}]_{\mathcal{K}\times\mathcal{I}}, [q_i]_{1\times\mathcal{I}})$,
  - where $n_{k,i}$ is the number of type-i patients in ward k, $i \in \mathcal{I}$, $k \in \mathcal{K}$.

- Action space:
  - The set of action space is denoted by $\mathcal{A}$,
  - A decision $a \in \mathcal{A}$ is defined as $a = ([x_{k,i}]_{\mathcal{K}\times\mathcal{I}}, [y_{k,\ell,i}]_{\mathcal{K}\times\mathcal{K}\times\mathcal{I}})$,
  - where $x_{k,i} \geq 0$ is allocation decision and $y_{k,\ell,i} \geq 0$ patients of type-i transferred from ward k to ward ℓ.

Transition probabilities

The transition from s_t to $s' = \left([n'_{k,i}]_{\mathcal{K}\times\mathcal{I}}, [q'_i]_{1\times\mathcal{I}}\right)$ is defined by:

$$\Pr\{s' | (s_t, a_t)\} = \prod_i \Pr(Q_i(t) = q_i) \prod_{k,i} \Pr(Z_{k,i}(t) = z_{k,i}) \qquad (2)$$

- where $\Pr(Q_i(t) = q_i) = \dfrac{(\lambda_i)^{q_i} e^{-\lambda_i}}{(q_i)!}, \forall i \in \mathcal{I}$.

Post-decision states

Making decision a_t while in state s_t will result in a post-decision $\bar{s}(n, q)$

$$\bar{n}_{k,i} = n_{k,i} + x_{k,i} + \sum_{k \in \mathcal{K}} y_{k,\ell,i} - \sum_{\ell \in \mathcal{K}} y_{k,\ell,i}. \quad (3)$$

where $x_{k,i}$ and $y_{k,\ell,i}$ are the decisions regarding the assignment and relocation of the patients, respectively.

The Optimality Equation

Using the value function $v : \mathcal{S} \rightarrow \mathbb{R}_{\mathbf{0}}^{+}$, we construct the Bellman's optimality equation

$$v(s) = \min_{a \in \mathcal{A}_s} \left\{ c(s, a) + \boldsymbol{\alpha} \sum_{s' \in \mathcal{S}} \Pr\{s' | (s, a)\} v(s') \right\} \forall s \in \mathcal{S}, \qquad (4)$$

where α is discount factor, and $c(s, a)$ is an immediate cost associate action $a \in \mathcal{A}_s$ in state $s \in \mathcal{S}$.

The Set of Feasible Action Space

$$\min\{\sum_{k\in\mathcal{K}} C_k, q_i\} \leq \sum_{k=1}^{K} x_{k,i} \ \forall i \quad (5)$$

where C_k is the available beds in ward k,

$$\sum_{\ell=1}^{K} y_{k,\ell,i} \leq n_{k,i} \quad (6)$$

$$\sum_{i=1}^{I}(x_{k,i} + n_{k,i}) + \sum_{\ell=1}^{K}\sum_{i=1}^{I}(y_{k,\ell,i} - y_{\ell,k,i}) \leq m_k \quad (7)$$

$$y_{k,\ell,i} \leq n_{k,i} \times d_{k,\ell,i}, \ y_{\ell,k,i} \leq n_{\ell,i} \times d_{\ell,k,,i}, \ d_{k,\ell,i} + d_{\ell,k,,i} \leq 1, \quad (8)$$

where $z_{\ell,k,,i} \in \{0,1\}$ is an auxiliary variable.

- The realistic size of this MDP is computationally intractable,
- We use Approximate Dynamic Programming techniques to resolve this issue,
- We use the linear programming form of $v(s)$ approximated by a set of basis functions similar to Gocgun and Puterman (2014) [1] ,
- A basis function $\Phi : S \to \mathbb{R}$ is set of features that contain state's information.

[1] Dynamic scheduling with due dates and time windows: an application to chemotherapy patient appointment booking. *Health Care Mange Sci*

We write the linear programming equivalent of Eq. (4) as

$$\max \sum_{s \in \mathcal{S}} \omega(s) v(s) \tag{9}$$

subject to

$$c(s, a) + \alpha \sum_{s' \in \mathcal{S}} \Pr\{s' | \, (s, a)\} v(s') \geq v(s) \; \forall a \in \mathcal{A}_s, \forall s \in \mathcal{S} \tag{10}$$

where $\omega(s)$ represent the weights of the states in (5).

- We approximate the LP using basis functions:

$$\bar{v}(s) = W_0 + \sum_{k=1}^{K} W_k \Phi_k(s) \tag{11}$$

  - where $\{W_k\}_{k=0}^{K} \in \mathbb{R}$ are weights that should be determined.

- We update the LP using this approximation

$$\max W_0 + \sum_{k=1}^{K} \left(\sum_{s \in \mathcal{S}} \omega(s) \Phi_k(s) \right) W_k \tag{12}$$

subject to

$$c(s,a) \geq (1-\alpha) W_0 + \sum_{k=1}^{K} v_k(s,a) W_k \;\forall a \in \mathcal{A}_s, \forall s \in \mathcal{S} \tag{13}$$

where

$$v_k(s,a) = \Phi_k(s) - \alpha \sum_{s' \in \mathcal{S}} \Pr\{s' |\, (s,a)\} \Phi_k(s') \geq v(s) \tag{14}$$

$$\forall a \in \mathcal{A}_s, \forall s \in \mathcal{S}$$

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Thank you!