Decision support model for patient admission scheduling problem with random arrivals and departures.

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Outline

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   - Patient flow in hospitals
   - Data for Benchmarking
   - Data structure

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   - Our Mathematical Model

3. Modelling random arrivals and departures

4. Solution approach

5. Conclusion
Patient admission scheduling (PAS) problem

- The PAS problem arises when patients arrive at the hospital.
- Patients need to be allocated to beds in an optimal manner.
- We need to take into account the availability of beds and the needs of patients.
- We consider this problem in a dynamic environment. That is, at the start of each day we record information about:
  - Registered patients (Known to the system),
  - Newly arrived (Emergency and planned patients),
  - Future arrivals (Planned patients).
- The goal is to determine optimal assignment of patients to rooms in order to minimise costs.
Patient flow in hospitals

- Planned patient
- Entrance
- Sign in
- Release
- Registration
- Treatment
- Transfer to room
- In-patient
- Release
- Emergence patient
- Entrance
- Release
50 instances were generated for each scenario by Ceschia et al. (2012).

<table>
<thead>
<tr>
<th>Patient family</th>
<th>Planning Horizon</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Short (14 days)</td>
<td>Medium (28 days)</td>
<td>Long (56 days)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Patients</td>
<td>Rooms</td>
<td>Patients</td>
<td>Rooms</td>
<td>Patients</td>
</tr>
<tr>
<td>Small</td>
<td>50</td>
<td>8</td>
<td>100</td>
<td>8</td>
<td>200</td>
</tr>
<tr>
<td>Medium</td>
<td>250</td>
<td>40</td>
<td>500</td>
<td>40</td>
<td>1000</td>
</tr>
<tr>
<td>Large</td>
<td>1000</td>
<td>160</td>
<td>2000</td>
<td>160</td>
<td>4000</td>
</tr>
</tbody>
</table>

Available at https://bitbucket.org/satt/pasu-instances.

Data was also used by Lusby et al. (2016).
Patient Admission Scheduling (PAS) Data structure

- **Departments:**
  - Specialisms (Cardiology, Dermatology, ...)
  - Set of rooms

- **Rooms:**
  - Beds
  - Features and Equipments: Oxygen, Telemetry, Infusion Pump, ...

- **Patients:**
  - Fixed arrival and departure dates
  - Requested treatments (related to specialisms)
  - Needed and preferred room features
  - Preferences on room capacity

- **Planning Horizon:**
  - Fixed number of days
Some of the notations we use are as follows;

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>An assignment</td>
</tr>
<tr>
<td>$x_{p,r,d}(\sigma)$</td>
<td>$\in {0, 1}$, is equal to 1 if a patient $p$ is assigned to a room $r$ on day $d$, 0 otherwise.</td>
</tr>
<tr>
<td>$y_{p,d}(\sigma)$</td>
<td>$\in {0, 1}$, is equal to 1 if a patient $p$ is admitted the hospital on day $d$, 0 otherwise.</td>
</tr>
<tr>
<td>$t_{p,r,r^*,d}(\sigma)$</td>
<td>$\in {0, 1}$, is equal to 1 if a patient $p$ is transferred from room $r$ to room $r^*$ on day $d$, 0 otherwise.</td>
</tr>
<tr>
<td>$Q_{r,d}(\sigma)$</td>
<td>$\in {0, 1}$, is equal to 1 if there is a gender conflict event observed in room $r$ on day $d$, 0 otherwise.</td>
</tr>
<tr>
<td>$Y_{r,d}(\sigma)$</td>
<td>Random variable records the number of patients in room $r$ on day $d$.</td>
</tr>
<tr>
<td>$d_p(\sigma)$</td>
<td>The admission date of a patient $p$.</td>
</tr>
<tr>
<td>$\ell_p(\sigma)$</td>
<td>The length of stay of a patient $p$.</td>
</tr>
<tr>
<td>$A_{m,d}(\sigma)$</td>
<td>The event that all males have left the room before day $d$.</td>
</tr>
<tr>
<td>$A_{f,d}(\sigma)$</td>
<td>The event that all females have left the room before day $d$.</td>
</tr>
</tbody>
</table>
We build our mathematical model on the model in Lusby et al. (2016).

Similarly to Lusby et al., we use **Integer Programming**.

Our contribution is a **stochastic model** which includes

- Random arrivals,
- Random departures,
- Stochastic objective function.
Objective function, component 1

The expected cost of \textit{assigning patients to rooms} is given by

\[
\sum_{p \in P} \sum_{d \in D} \sum_{r \in R} c_{p,r} \times x_{p,r,d}(\sigma) \times Pr(L_p \geq d - d_p(\sigma)). \quad (1)
\]

- $c_{p,r}$ is the cost of assigning patient $p$ to a room $r$.
- $L_p$ is the random variable records the length of stay of $p$ till discharge.
- $d_p(\sigma)$ is the admission day.
The expected cost of all transfers is given by

$$\sum_{p \in \mathcal{P}} \sum_{d \in \mathcal{D}} \sum_{r \in \mathcal{R}} c_{p,r,r^*}^{(T)} \times t_{p,r,r^*,d}(\sigma) \times \Pr(L_p \geq d - d_p(\sigma)). \quad (2)$$

- $c_{p,r,r^*}^{(T)}$ is the cost of transferring patient $p$ from room $r$ to room $r^*$ on day $d$.
- $c_{p,r,r^*}^{(T)} = 0$, when $r = r^*$. 
The total expected penalty for all gender violations is given by

\[ \sum_{d \in \mathcal{D}} \sum_{r \in \mathcal{R}} c^{(G)}_{r,d} \times P_r(Q_{r,d}(\sigma)). \]  

- \( c^{(G)}_{r,d} \) is the penalty for gender violation for room \( r \) on day \( d \).
- \( Q_{r,d}(\sigma) \) is an event that records gender conflict in room \( r \) on day \( d \).
We calculate $P_r(Q_{r,d}(\sigma))$ as follows:

\[
1 - P_r(Q_{r,d}(\sigma)) = P_r(A_{m,d}(\sigma)) + P_r(A_{f,d}(\sigma)) \\
- P_r(A_{m,d}(\sigma) \cap A_{f,d}(\sigma)) \\
= \prod_{M_{r,d}} x_{p,r,d}(\sigma)P_r(L_p < d - d_p(\sigma)) \\
+ \prod_{F_{r,d}} x_{p,r,d}(\sigma)P_r(L_p < d - d_p(\sigma)) \\
- \prod_{M_{r,d} \cup F_{r,d}} x_{p,r,d}(\sigma)P_r(L_p < d - d_p(\sigma)).
\]

(4)

- $F_{r,d}$ is the set of all female patients assigned to room $r$ on day $d$.
- $M_{r,d}$ is the set of all male patients assigned to room $r$ on day $d$. 
The total expected penalty for **overcrowding** is given by

\[
\sum_{d \in \mathcal{D}} \sum_{r \in \mathcal{R}} c^{(O)}_{r,d} \times \left( \max\{0, \frac{E(Y_{r,d}(\sigma)) - \kappa_r}{\hat{\kappa}_r - \kappa_r}\} \right). \tag{5}
\]

- \(c^{(O)}_{r,d}\) is the penalty for overcrowding in room \(r\) on day \(d\).
- \(Y_{r,d}(\sigma)\) is a random variable recording the number of patients in room \(r\) on day \(d\), given assignment \(\sigma\).
- \(\kappa_r\) is the capacity of room \(r\).
- \(\hat{\kappa}_r\) is the maximum room capacity of room \(r\), \((\hat{\kappa}_r > \kappa_r)\).
- \(\hat{\kappa}_r - \kappa_r\) is a threshold.
(E(Y_{r,d}(\sigma)) - \kappa_r) is the expected excess in room r on day d. We then have

\[
E(Y_{r,d}(\sigma)) = E\left(\sum_{p \in \mathcal{P}} Z_{p,r,d}(\sigma)\right)
\]

\[
= \sum_{p \in \mathcal{P}} E \left(Z_{p,r,d}(\sigma)\right)
\]

\[
= \sum_{p \in \mathcal{P}} P_r \left(Z_{p,r,d}(\sigma) = 1\right)
\]

\[
= \sum_{p \in \mathcal{P}} x_{p,r,d}(\sigma) P_r(L_p \geq d - d_p(\sigma)). \tag{6}
\]

\[
Z_{p,r,d}(\sigma) = \begin{cases} 
1 & \text{if patient } p \text{ is in room } r \text{ on day } d \\
0 & \text{otherwise},
\end{cases}
\]
Objective function, component 5

The total expected penalty for admission delay is given by

\[ \sum_{p \in \mathcal{P}} c_{p,d}^{(De)} \times \sum_{d \in \mathcal{D}} \left( \frac{d - d_{p}^{plan}}{d_{p}^{max} - d_{p}^{plan}} \right) \times y_{p,d}(\sigma). \] (7)

- \( c_{p,d}^{(De)} \) is the penalty for admission delay on day \( d \).
- \( d - d_{p}^{plan} \) is the admission delay for patient \( p \) on day \( d \).
- \( y_{p,d}(\sigma) = \begin{cases} 1 & \text{if patient } p \text{ is admitted on day } d \\ 0 & \text{otherwise,} \end{cases} \)
- \( d_{p}^{max} - d_{p}^{plan} \) is the maximum delay.
Hard constraints

- **Room capacity** ($\tilde{\kappa}_r$): the number of patients assigned to a room must be less than the maximum room capacity.
  \[
  \sum_{p \in P} x_{p,r,d}(\sigma) \leq \tilde{\kappa}_r, \quad \forall r \in R, \forall d \in D. \tag{8}
  \]

- **The Patient age** ($A_p$): has to be within the minimum, and maximum age limit policy of the ward.
  \[
  \begin{align*}
  x_{p,r,d}(\sigma) & \leq A_p & \forall p \in \mathcal{P}, \forall r \in W_i, \forall d \in D. & \tag{9} \\
  a(W_i) & \leq A_p & \forall p \in \mathcal{P}, \forall r \in W_i, \forall d \in D. & \tag{10} \\
  A(W_i) & \geq A_p & \forall p \in \mathcal{P}, \forall r \in W_i, \forall d \in D. & \tag{11}
  \end{align*}
  \]

- $a(W_i)$ minimum, and $A(W_i)$ maximum age limit in ward $W_i$. 
Hard constraints

- **Patient admission** \((y_{p,d}(\sigma))\): a patient must be admitted on day \(d\) to be assigned to a room.

\[
\sum_{d \in D_p} y_{p,d}(\sigma) = 1, \quad \forall p \in P. \tag{12}
\]

- Patients should stay in the room \((r)\) the following \((\ell_p - 1)\) nights

\[
\sum_{r \in R} x_{p,r,d}(\sigma) \geq y_{p,\bar{d}(\sigma)}, \quad \forall p \in P, \bar{d}_p \in D_p, \bar{d} = \bar{d}_p, \ldots, \bar{d}_p + \ell_p - 1. \tag{13}
\]
Soft constraints

- **Room gender**: calculates the presence of female patients ($f_{r,d}(\sigma)$), male patients ($m_{r,d}(\sigma)$), or both ($b_{r,d}(\sigma)$) in a room $r$ on day $d$.

  \[
  f_{r,d}(\sigma) \geq x_{p,r,d}(\sigma), \quad \forall p \in \mathcal{F}, \forall r \in \mathcal{R} \cap \mathcal{R}^{SG}, \forall d \in \mathcal{D}. \quad (14)
  \]

  - $f_{r,d}(\sigma) = \begin{cases} 
  1 & \text{if there is at least one female in room } r \text{ on day } d, \\
  0 & \text{otherwise.}
  \end{cases}$

  \[
  m_{r,d}(\sigma) \geq x_{p,r,d}(\sigma), \quad \forall p \in \mathcal{M}, \forall r \in \mathcal{R} \cap \mathcal{R}^{SG}, \forall d \in \mathcal{D}. \quad (15)
  \]

  - $m_{r,d}(\sigma) = \begin{cases} 
  1 & \text{if there is at least one male in room } r \text{ on day } d, \\
  0 & \text{otherwise.}
  \end{cases}$
Soft constraints

- **Room gender (continued)**

  \[ b_{r,d}(\sigma) \geq m_{r,d} + f_{r,d}(\sigma) - 1, \quad \forall r \in R^{SG}, \forall d \in D. \]  
  (16)

  \[ b_{r,d}(\sigma) = \begin{cases} 
  1 & \text{if both genders are present in room } r \text{ on day } d, \\
  0 & \text{otherwise.} 
\end{cases} \]

- **Transfer**: captures the number of transfer from room \( r \) to room \( r^* \).

  \[ t_{p,r,r^*,d}(\sigma) \geq x_{p,r,d}(\sigma) - x_{p,r,d-1}(\sigma) - y_{p,d}(\sigma), \]  
  (17)

  \[ \forall p \in P, \forall r \in R, d = 2, \ldots, D. \]
Random departures

- \( L_p \) is a random variable that records the LoS of the type-\( p \) patient,
- Takes values \( \ell_p = 0, 1, \ldots, \ell_p^{\text{max}} \), for some positive integer \( \ell_p^{\text{max}} \),
- We consider a discrete-time Markov chain distribution.
- State space \( \mathcal{V} = \{0, 1, \ldots, \ell_p^{\text{max}}\} \), where \( \ell_p^{\text{max}} \) is an absorbing state, and one-step transition probability matrix \( P \) given by

\[
P^* = \begin{bmatrix} P & p \\ 0 & 1 \end{bmatrix}
\]

(Example: Coxian phase type distribution)
for some matrix $P = [P_{ij}]_{i,j=0,1,...,\ell^p_{\max} - 1}$ and (column) vector $p = [p_{i\ell^p_{\max}}]_{j=0,1,...,\ell^p_{\max} - 1}$, and the initial distribution (row) vector $\tau = [\tau_i]_{i=0,1,...,\ell^p_{\max} - 1}$.

We then assume that the random variable $L_p$ follows **discrete phase-type distribution** with parameters $\tau$ and $P$, which models time till absorption in the above chain,

$$L_p \sim PH(\tau, P),$$

which gives, for $\ell_p = 0, 1, \ldots, \ell^p_{\max}$,

$$Pr(L_p = \ell_p) = \tau P^{\ell_p} p,$$

$$Pr(L_p \leq \ell_p) = 1 - \tau P^{\ell_p} 1,$$
We use similar technique to Kumar et al. (2018) to include the random arrivals,

- We simulate random arrivals (from a suitable distribution) multiple times, resulting in a number of possible solutions.
- We then compare the different solutions by running simulations over some long time period, and then choose the preferred solution.
We use **simulation** to generate random inputs for our model.

Apply **Meta heuristic algorithms**, such as

- Greedy search
- Adaptive neighbourhood search and
- Simulated annealing

to solve the stochastic integer program.

**Compare** our results with Lusby et al. (2016).
Current and future work

- Code for numerical examples based on the model in Lusby et al. (2016).
- Code for numerical examples based our mathematical model.
- Use solution based on the model in Lusby et al. (2016) as the initial solution to our mathematical model.
- Comparison of the results.
Doctor Waiting Room

If you die whilst waiting to see the doctor please cancel your appointment


Thank you!