Decision support model for patient admission scheduling problem with random arrivals and departures.

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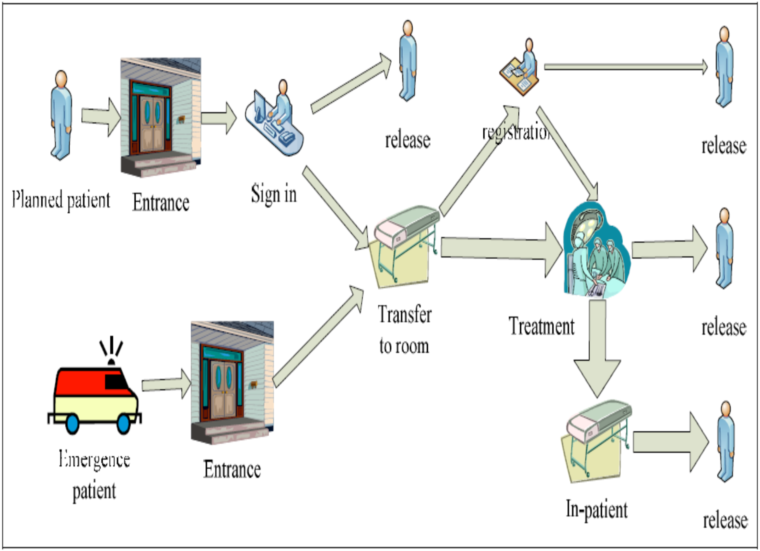
Outline

Patient admission scheduling (PAS) problem

- The PAS problem arises when patients arrive at the hospital.
- Patients need to be allocated to beds in an optimal manner.
- We need to take into account the availability of beds and the needs of patients.
- We consider this problem in a dynamic environment. That is, at the start of each day we record information about:
  - Registered patients (Known to the system),
  - Newly arrived (Emergency and planned patients),
  - Future arrivals (Planned patients).
- The goal is to determine optimal assignment of patients to rooms in order to minimise costs.

- Patient flow in hospitals



- 50 instances were generated for each scenario by Ceschia et al. (2012).

Patient family	Planning Horizon					
	Short (14 days)		Medium (28 days)		Long (56 days)	
	Patients	Rooms	Patients	Rooms	Patients	Rooms
Small	50	8	100	8	200	8
Medium	250	40	500	40	1000	40
Large	1000	160	2000	160	4000	160

- Available at https://bitbucket.org/satt/pasu-instances.

- Data was also used by Lusby et al. (2016).

- Departments:
  - Specialisms (Cardiology, Dermatology, . . .)
  - Set of rooms
- Rooms:
  - Beds
  - Features and Equipments: Oxygen, Telemetry, Infusion Pump, . . .
- Patients:
  - Fixed arrival and departure dates
  - Requested treatments (related to specialisms)
  - Needed and preferred room features
  - Preferences on room capacity
- Planning Horizon:
  - Fixed number of days

- Some of the notations we use are as follows;

Notation	**Description**
σ	An assignment
$x_{p,r,d}(\sigma)$	$\in \{0,1\}$, is equal to 1 if a patient p is assigned to a room r on day d, 0 otherwise.
$y_{p,d}(\sigma)$	$\in \{0,1\}$, is equal to 1 if a patient p is admitted the hospital on day d, 0 otherwise.
$t_{p,r,r^*,d}(\sigma)$	$\in \{0,1\}$, is equal to 1 if a patient p is transferred from room r to room r^* on day d, 0 otherwise.
$Q_{r,d}(\sigma)$	$\in \{0,1\}$, is equal to 1 if there is a gender conflict event observed in room r on day d, 0 otherwise.
$Y_{r,d}(\sigma)$	Random variable records the number of patients in room r on day d.
$d_p(\sigma)$	The admission date of a patient p.
$\ell_p(\sigma)$	The length of stay of a patient p.
$A_{m,d}(\sigma)$	The event that all males have left the room before day d.
$A_{f,d}(\sigma)$	The event that all females have left the room before day d.

- We build our mathematical model on the model in Lusby et al. (2016).
- Similarly to Lusby et al., we use Integer Programming.
- Our contribution is a stochastic model which includes
  - Random arrivals,
  - Random departures,
  - Stochastic objective function.

Objective function, component 1

- The expected cost of assigning patients to rooms is given by

$$\sum_{p \in \mathcal{P}} \sum_{d \in \mathcal{D}} \sum_{r \in \mathcal{R}} c_{p,r} \times x_{p,r,d}(\sigma) \times Pr(L_p \geq d - d_p(\sigma)). \qquad (1)$$

  - $C_{p,r}$ is the cost of assigning patient p to a room r.
  - L_p is the random variable records the length of stay of p till discharge.
  - $d_p(\sigma)$ is the admission day.

Objective function, component 2

- The expected cost of all transfers is given by

$$\sum_{p\in\mathcal{P}}\sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}} c^{(T)}_{p,r,r^*} \times t_{p,r,r^*,d}(\sigma) \times Pr(L_p \geq d - d_p(\sigma)). \qquad (2)$$

  - $c^{(T)}_{p,r,r^*}$ is the cost of transferring patient p from room r to room r^* on day d.
  - $c^{(T)}_{p,r,r^*} = 0$, when $r = r*$.

Objective function, component 3

- The total expected penalty for all gender violations is given by

$$\sum_{d \in \mathcal{D}} \sum_{r \in \mathcal{R}} c_{r,d}^{(G)} \times P_r(Q_{r,d}(\sigma)). \tag{3}$$

  - $c_{r,d}^{(G)}$ is the penalty for gender violation for room r on day d.
  - $Q_{r,d}(\sigma)$ is an event that records gender conflict in room r on day d.

continued

- We calculate $P_r(Q_{r,d}(\sigma))$ as follows;

$$
\begin{aligned}
1 - P_r(Q_{r,d}(\sigma)) = {} & P_r(A_{m,d}(\sigma)) + P_r(A_{f,d}(\sigma)) \\
& - P_r(A_{m,d}(\sigma) \cap A_{f,d}(\sigma)) \\
= {} & \prod_{\mathcal{M}_{r,d}} x_{p,r,d}(\sigma) P_r(L_p < d - d_p(\sigma)) \\
& + \prod_{\mathcal{F}_{r,d}} x_{p,r,d}(\sigma) P_r(L_p < d - d_p(\sigma)) \\
& - \prod_{\mathcal{M}_{r,d} \cup \mathcal{F}_{r,d}} x_{p,r,d}(\sigma) P_r(L_p < d - d_p(\sigma)).
\end{aligned}
\tag{4}
$$

- $\mathcal{F}_{r,d}$ is the set of all female patients assigned to room r on day d.
- $\mathcal{M}_{r,d}$ is the set of all male patients assigned to room r on day d.

Objective function, component 4

- The total expected penalty for overcrowding is given by

$$\sum_{d\in\mathcal{D}}\sum_{r\in\mathcal{R}} c^{(O)}_{r,d} \times \left(\frac{max\{0,\ E(Y_{r,d}(\sigma)) - \kappa_r\}}{\widehat{\kappa}_r - \kappa_r}\right). \qquad (5)$$

  - $c^{(O)}_{r,d}$ is the penalty for overcrowding in room r on day d.
  - $Y_{r,d}(\sigma)$ is a random variable recording the number of patients in room r on day d, given assignment σ.
  - κ_r is the capacity of room r.
  - $\widehat{\kappa}_r$ is the maximum room capacity of room r, ($\widehat{\kappa}_r > \kappa_r$).
  - $\widehat{\kappa}_r - \kappa_r$ is a threshold.

continued

- $(E(Y_{r,d}(\sigma)) - \kappa_r)$ is the expected excess in room r on day d. We then have

$$\begin{aligned} E(Y_{r,d}(\sigma)) &= E\left(\sum_{p\in\mathcal{P}} Z_{p,r,d}(\sigma)\right) \\ &= \sum_{p\in\mathcal{P}} E\left(Z_{p,r,d}(\sigma)\right) \\ &= \sum_{p\in\mathcal{P}} P_r\left(Z_{p,r,d}(\sigma) = 1\right) \\ &= \sum_{p\in\mathcal{P}} x_{p,r,d}(\sigma) P_r(L_p \geq d - d_p(\sigma)). \end{aligned} \tag{6}$$

- $Z_{p,r,d}(\sigma) = \begin{cases} 1 & \text{if patient } p \text{ is in room } r \text{ on day } d \\ 0 & \text{otherwise,} \end{cases}$

Objective function, component 5

- The total expected penalty for admission delay is given by

$$\sum_{p\in\mathcal{P}} c_{p,d}^{(De)} \times \sum_{d\in\mathcal{D}} \left(\frac{d - d_p^{plan}}{d_p^{max} - d_p^{plan}} \right) \times y_{p,d}(\sigma). \tag{7}$$

  - $c_{p,d}^{(De)}$ is the penalty for admission delay on day d.
  - $d - d_p^{plan}$ is the admission delay for patient p on day d.
  - $y_{p,d}(\sigma) = \begin{cases} 1 & \text{if patient } p \text{ is admitted on day } d \\ 0 & \text{otherwise,} \end{cases}$
  - $d_p^{max} - d_p^{plan}$ is the maximum delay.

Hard constraints

- **Room capacity ($\widehat{\kappa}_r$)**: the number of patients assigned to a room must be less than the maximum room capacity.

$$\sum_{p \in P} x_{p,r,d}(\sigma) \leq \widehat{\kappa}_r, \quad \forall r \in R, \forall d \in \mathcal{D}. \tag{8}$$

- **The Patient age (A_p)**: has to be within the minimum, and maximum age limit policy of the ward.

$$x_{p,r,d}(\sigma) \leq A_p \quad \forall p \in \mathcal{P}, \forall r \in W_i, \forall d \in \mathcal{D}. \tag{9}$$

$$a(W_i) \leq A_p \quad \forall p \in \mathcal{P}, \forall r \in W_i, \forall d \in \mathcal{D}. \tag{10}$$

$$A(W_i) \geq A_p \quad \forall p \in \mathcal{P}, \forall r \in W_i, \forall d \in \mathcal{D}. \tag{11}$$

  - $a(W_i)$ minimum, and $A(W_i)$ maximum age limit in ward W_i.

Hard constraints

- **Patient admission ($y_{p,d}(\sigma)$)**: a patient must be admitted on day d to be assigned to a room.

$$\sum_{d \in D_p} y_{p,d}(\sigma) = 1, \quad \forall p \in P. \tag{12}$$

- Patients should stay in the room (r) the following ($\ell_p - 1$) nights

$$\sum_{r \in R} x_{p,r,d}(\sigma) \geq y_{p,\bar{d}}(\sigma), \forall p \in P, \bar{d}_p \in D_p, d = \bar{d}_p, \ldots, \bar{d}_p + \ell_p - 1. \tag{13}$$

Soft constraints

- **Room gender**: calculates the presence of female patients ($f_{r,d}(\sigma)$), male patients ($m_{r,d}(\sigma)$), or both ($b_{r,d}(\sigma)$) in a room r on day d.

$$f_{r,d}(\sigma) \geq x_{p,r,d}(\sigma), \quad \forall p \in \mathcal{F}, \forall r \in \mathcal{R} \cap R^{SG}, \forall d \in \mathcal{D}. \tag{14}$$

  - $f_{r,d}(\sigma) = \begin{cases} 1 & \text{if there is at least one female in room } r \text{ on day } d, \\ 0 & \text{otherwise.} \end{cases}$

$$m_{r,d}(\sigma) \geq x_{p,r,d}(\sigma), \quad \forall p \in \mathcal{M}, \forall r \in \mathcal{R} \cap R^{SG}, \forall d \in \mathcal{D}. \tag{15}$$

  - $m_{r,d}(\sigma) = \begin{cases} 1 & \text{if there is at least one male in room } r \text{ on day } d, \\ 0 & \text{otherwise.} \end{cases}$

Soft constraints

- Room gender (continued)

$$b_{r,d}(\sigma) \geq m_{r,d} + f_{r,d}(\sigma) - 1, \quad \forall r \in R^{SG}, \forall d \in \mathcal{D}. \tag{16}$$

  - $b_{r,d}(\sigma) = \begin{cases} 1 & \text{if both genders are present in room } r \text{ on day } d, \\ 0 & \text{otherwise.} \end{cases}$

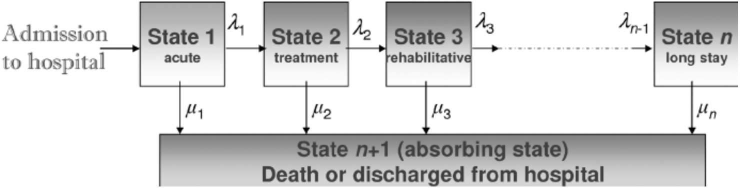
- **Transfer**: captures the number of transfer from room r to room r^*.

$$t_{p,r,r^*,d}(\sigma) \geq x_{p,r,d}(\sigma) - x_{p,r,d-1}(\sigma) - y_{p,d}(\sigma), \quad \forall p \in P, \forall r \in R, d = 2, \ldots, \mathcal{D}. \tag{17}$$

Random departures

- L_p is a random variable that records the LoS of the type-p patient,
- Takes values $\ell_p = 0, 1, \ldots, \ell_p^{max}$, for some positive integer ℓ_p^{max},
- We consider a discrete-time Markov chain distribution.
- State space $\mathcal{V} = \{0, 1, \ldots, \ell_p^{max}\}$, where ℓ_p^{max} is an **absorbing state**, and one-step transition probability matrix **P** given by

$$\mathbf{P}^* = \begin{bmatrix} \mathbf{P} & \mathbf{p} \\ \mathbf{0} & 1 \end{bmatrix}$$



(Example: Coxian phase type distribution)

- for some matrix $\mathbf{P} = [P_{ij}]_{i,j=0,1,\ldots,\ell_p^{max}-1}$ and (column) vector $\mathbf{p} = [p_{i\ell_p^{max}}]_{j=0,1,\ldots,\ell_p^{max}-1}$, and the initial distribution (row) vector $\tau = [\tau_i]_{i=0,1,\ldots,\ell_p^{max}-1}$.
- We then assume that the random variable L_p follows **discrete phase-type distribution** with parameters τ and $\mathbf{P}$, which models time till absorption in the above chain,

$$L_p \sim PH(\tau, \mathbf{P}),$$

which gives, for $\ell_p = 0, 1, \ldots, \ell_p^{max}$,

$$\begin{aligned} Pr(L_p = \ell_p) &= \tau \mathbf{P}^{\ell_p} \mathbf{p}, \\ Pr(L_p \leq \ell_p) &= 1 - \tau \mathbf{P}^{\ell_p} \mathbf{1}, \end{aligned} \tag{18}$$

Random arrivals

- We use similar technique to Kumar et al. (2018) to include the random arrivals,
  - We simulate random arrivals (from a suitable distribution) multiple times, resulting in a number of possible solutions.
  - We then compare the different solutions by running simulations over some long time period, and then choose the preferred solution.

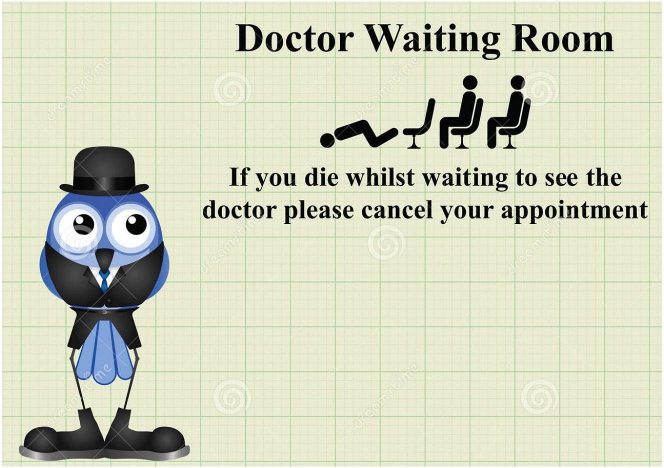
Method

- We use **simulation** to generate random inputs for our model.
- Apply **Meta heuristic algorithms**, such as
  - Greedy search
  - Adaptive neighbourhood search and
  - Simulated annealing

  to solve the stochastic integer program.
- **Compare** our results with Lusby et al. (2016).

Current and future work

- Code for numerical examples based on the model in Lusby et al. (2016).
- Code for numerical examples based our mathematical model.
- Use solution based on the model in Lusby et al. (2016) as the initial solution to our mathematical model.
- Comparison of the results.


Doctor Waiting Room
If you die whilst waiting to see the
doctor please cancel your appointment

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Thank you!