On some fixed-point problems connecting branching and queueing

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Abstract

Connections between branching and queueing have a long history. A classical case is the M/G/1 queue where one can view the children of a customer as the say $N$ customers arriving during his service time $S$. This leads to the fixed-point equation

$$ R \overset{d}{=} S + \sum_{i=1}^{N} R_i $$

(1)

for the busy period $R$, where $R_1, R_2, \ldots$ are i.i.d. and independent of $(S, N)$. Similar equations occur in other branching process connections. A quick application is to note that the queue is stable if and only if the corresponding branching process is subcritical, which immediately gives the $\rho < 1$ criterion. Equations similar have been used in recent work ([5], [9]) related to the Google page rank algorithm to derive tail asymptotics of $R$ under regular variation (RV); for the busy period, the RV asymptotics has earlier been studied in [6] and [10]. We present a simple random walk argument from [1] to give a short proof of these results as well as certain extensions. Motivated from a multiclass queueing model originating from [4], also a multivariate version of (1) is studied under RV conditions.

Following [2], we also consider preemptive-repeat LIFO queues where the time $R$ in system is related to the fixed-point equation

$$ R(s) \overset{d}{=} T \land s + 1(T \leq s)(R + R^*(s)) $$

(2)

where $T$ is the interarrival time and $R(s)$ the time-in-system of a customer with service time $s$. Using again a branching connection gives a highly non-standard stability condition for the M/G/1 case. However, for GI/G/1 equation (2) does not have the correct interpretation, and we present a matrix-analytic approach that lead to an algorithm for finding the stability region for PH/G/1. The approach indeed uses a connection to a (multitype) branching process, but meets the difficulty that the offspring distribution is not explicit. For somewhat related models, MAM have earlier been used in [7] and [3].

References


