

# Rate of strong convergence of stochastic fluid processes to Markov-modulated Brownian motion

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## ABSTRACT

In [2], the authors constructed a sequence of stochastic fluid processes and showed that it converges weakly to a Markov-modulated Brownian motion  $\{(X_t, \varphi_t)\}_{t \geq 0}$ .

Here, we construct another sequence of stochastic fluid processes, with different characteristics to the ones considered in [2], and show that it converges strongly to  $\{(X_t, \varphi_t)\}_{t \geq 0}$ . We also show that the rate of this almost sure convergence is proportional to  $n^{-1/2} \log n$ .

When reduced to the special case of standard Brownian motion, our convergence rate is an improvement over that obtained by [1], which is proportional to  $n^{-1/2}(\log n)^{5/2}$ .

## 1. REFERENCES

- [1] L. G. Gorostiza and R. J. Griego. Rate of convergence of uniform transport processes to Brownian motion and application to stochastic integrals. *Stochastics*, 3(1-4):291–303, 1980.
- [2] G. Latouche and G. T. Nguyen. The morphing of fluid queues into Markov-modulated Brownian motion. *Stochastic Systems*, 5(1):62–86, 2015.