KMA354 Partial Differential Equations

Large Assignment. Due last week of Semester 2, 2006.

Task

In groups of two, you are required to research a PDE or special 2nd order linear ODE, finding information about it and its solutions. Properties to consider might be (but not limited to, nor necessarily including all of)

- general solutions and their graphs,
- relations between solutions,
- recurrence relations,
- interesting representations (perhaps integral rather than series) for the solutions,
- relationships between the solutions and other special functions,
- derivatives of the solutions,
- is the equation self-adjoint; and what are the coefficients, integrating factor ... ?
- differences between integer and non-integer order solutions?

In addition to finding general mathematical information, you must also find what areas the nominated equation typically finds application in. This may be as simple as reviewing an article in which the nominated equation has been used.

Assessment

The assignment will be assessed in two ways: (i) a written report, and (ii) a 5-10 minute oral presentation.

The written report must not be a print-out of your oral presentation - it should be a document that can stand on its own.

The mode of presentation is your choosing. Recommended modes are PowerPoint display, Mathematica display, LaTeX slide show, or overhead transparencies. Writing on the white board is not recommended as this can be very time consuming. The presentation should be something like a dot-point summary of your report, where a dot-point serves as a lead-in to a more detailed discussion.

Presentations will be scheduled for the Wednesday and Thursday lecture and tutorial slots in the final week of semester.

This assignment contributes 10% of your final mark for the unit. The written/oral components will be split 60/40. The written component will be assessed by me, but presentations will be assessed by the class as well as me. You will be required to submit a rough estimate of your partner's relative contribution to the assignment overall. Should there be an uneven contribution, marks may be weighted accordingly.

Pairings and Topic

The pairings below are the results of a Mathematica notebook scripting a random allocation of pairs. These pairings are set in stone. The topics are also randomly allocated but are not set in stone. If you have a PDE or ODE in mind that you would prefer to research, suggest it to me and I'll consider it. On the last page of this assignment sheet are some tables with a small selection of ODEs and PDEs - this list is not exhaustive.

Alexander Page	Chris Jackett	Associated Laguerre
Jonathon van Rossum	Matthew O'Brien	Hermite
Onn Ping Chin	Daniel Segovia	Mathieu
Jeremy D'Alton	Luke Rybarczyk	Airy
Thomas Lorimer	Tristan Brookes	Chebyshev (Tchebycheff)
Michael Ford	James Gill	Struve
Benjamin Lewis	James Jackson	Hypergeometric
Shaun Inglis	Kate Rutherford	Modified Spherical Bessel
Mark Lake	Jeremy Breen	Weber

Reference Material

Books such as

- *CRC standard mathematical tables and formulae*, Zwillinger D.
- *Table of Integrals, Series, and Products*, Gradshteyn I. S. and Ryzhik I. M.
- *Handbook of Mathematical Functions*, Abramowitz M. and Stegun, I.

are excellent sources of information for many of the listed ODEs and their respective solutions. Many of these titles can be found in the Reference section of the Science Library in the vicinity of QA47. Literature can also be found in the Science Library catalogued in the areas QA40–QA47, QA331, and Q351. There are other areas but I'm not going to list them all for you.

The internet is an excellent source of information, with many sites referencing the texts listed above. Try

- Wikipedia (http://en.wikipedia.org/wiki/Partial_differential_equation)
- Digital Library of Mathematical Functions (http://dlmf.nist.gov/Draft/TheIndex.php)
- PlanetMath (http://planetmath.org)
- Mathworld (http://mathworld.wolfram.com/topics/OrdinaryDifferentialEquations.html)

or search with Google or some other search engine.

For article searches, use the University Library's Database option. A link to the databases is available from the front page of their online catalogue, or use this URL: http://www.utas.edu.au/library/info/dbase/index.html. Databases that may be useful are

- Inspec
- Current Contents
- Web of Science
- MathSci
- Proquest

Examples of PDEs

Ito Calculus	stochastic processes - Financial mathematics, Brownian motion, ...
Brown-Scholes (Options pricing)	$rF - rx\frac{\partial F}{\partial x} - \frac{1}{2}\,v^2x^2\frac{\partial^2 F}{\partial x^2} = \frac{\partial F}{\partial t}$
Biharmonic Equation	$\nabla^4 F = 0$
Tricomi Equation	$\frac{\partial^2 F}{\partial y^2} = y\frac{\partial^2 F}{\partial x^2}$
Korteweg - de Vries Equation	$\frac{\partial F}{\partial t} + \frac{\partial^3 F}{\partial x^3} - 6F\frac{\partial F}{\partial x} = 0$

Examples of ODEs

Modified Bessel	$x^2\frac{d^2X}{dx^2} + x\frac{dX}{dx} - (x^2+\nu^2)X = 0$
Hypergeometric	$x(1-x)\frac{d^2X}{dx^2} + [c-(a+b+1)x]\frac{dX}{dx} - a\,b\,X = 0$
Chebyshev (Tchebycheff)	$(1-x^2)\frac{d^2X}{dx^2} - x\frac{dX}{dx} + \alpha^2 X = 0$
Associated Laguerre	$x\frac{d^2X}{dx^2} + (\nu+1-x)\frac{dX}{dx} + \lambda\,X = 0$
Coulomb Wave	$\frac{d^2X}{dx^2} + \left[1 - \frac{2\nu}{x} - \frac{l(l+1)}{x^2}\right]X = 0$
Gegenbauer	$(1-x^2)\frac{d^2X}{dx^2} - 2(\mu+1)x\frac{dX}{dx} + (\nu-\mu)(\nu+\mu+1)X = 0$
Associated Legendre	$(1-x^2)\frac{d^2X}{dx^2} - 2x\frac{dX}{dx} + \left[l(l+1) - \frac{m^2}{1-x^2}\right]X = 0$
Hermite	$\frac{d^2X}{dx^2} - 2x\frac{dX}{dx} + 2n\,X = 0$
Mathieu	$\frac{d^2X}{dx^2} + [a - 2q\cos(2x)]X = 0$
Weber	$\frac{d^2X}{dx^2} + (n + \frac{1}{2} - \frac{1}{4}x^2)X = 0$
Airy	$\frac{d^2X}{dx^2} \pm k^2 x\,X = 0$
Lamé	$(x^2-b^2)(x^2-c^2)\frac{d^2X}{dx^2} + x(2x^2-b^2-c^2)\frac{dX}{dx} - [m(m+1)x^2 - (b^2+c^2)p]X = 0$
Struve	$x^2\frac{d^2X}{dx^2} + x\frac{dX}{dx} + (x^2-\nu^2)X = \frac{4(\frac{1}{2}x)^{\nu+1}}{\sqrt{\pi}\,\Gamma(\nu+\frac{1}{2})}$
Modified Spherical Bessel	$x^2\frac{d^2X}{dx^2} + 2x\frac{dX}{dx} - (x^2+n(n+1))X = 0$