Learning Outcomes: This assignment will give you practice in

- the computation of row echelon forms in relation to Gaussian elimination;
- noting that familiar results from numerical arithmetic can’t be assumed to hold for matrix arithmetic;
- constructing a Taylor series approximation and determining conditions on its convergence and accuracy;
- assessing the limits of an indeterminate function by approximating it as a power series.

1. (i) Transform the matrix
\[
\begin{bmatrix}
2 & 2 & 0 & 0 & -4 \\
0 & 1 & 0 & -3 & -9 \\
0 & 0 & 0 & 4 & 12 \\
3 & 3 & 1 & 1 & -3
\end{bmatrix}
\]
to row reduced echelon form.

(ii) DEDUCE the solution(s) to the systems
\[
\begin{align*}
2x_1 + 2x_2 &= -4 \\
x_2 - 3x_4 &= -9 \\
4x_4 &= 12 \\
3x_1 + 3x_2 + x_3 + x_4 &= -3
\end{align*}
\]
and
\[
\begin{align*}
2x_1 + 2x_2 - 4x_5 &= 0 \\
x_2 - 3x_4 - 9x_5 &= 0 \\
4x_4 + 12x_5 &= 0 \\
3x_1 + 3x_2 + x_3 + x_4 - 3x_5 &= 0
\end{align*}
\]

2. (i) If $A, B$ are matrices such that $AB = A + B$ and $A$ is a $2 \times k$ matrix, what is $k$?

(ii) Let $C, D$ be $n \times n$ matrices such that $C^2 - D^2 = (C + D)(C - D)$. Show that $CD = DC$.

(Hence the “difference of two squares formula” is false for matrices)

3. Find the Taylor series for $f(x) = \ln x$ about $a = 5$. Find the radius of convergence of this series. Use Taylor’s formula to prove that the remainder $R_n(x) \to 0$ when $|x - a| \leq 1$.

4. The sinc function is defined as
The special definition at $x = 0$ is required else an indeterminate solution would result. Use the following Maclaurin series approximation: $\sin x \approx \sum_{m=0}^{2} \frac{(-1)^m}{(2m + 1)!} x^{2m+1}$, to explain why $\lim_{x \to 0} \frac{\sin x}{x} = 1$.

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Problems 8

For ‘Problem Solving’ session Tuesday September 19, 2006

1. Let $f(x) = \frac{1}{\sqrt{1 + x^3}}$.

Show that $f^{(9)}(0) = -\frac{9!}{16}$.

2. Prove that $e$ is an irrational number.

Hint: Think in terms of power series (like Taylor, Maclaurin, and Binomial series).